The Virtue of Complexity

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“Principle of Parsimony” (Tukey, 1961)

Textbook Rule #1

> “It is important, in practice, that we employ the **smallest possible** number of parameters for adequate representations” (Box and Jenkins, *Time Series Analysis: Forecasting and Control*)

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- Leading LLMs (e.g., GPT) use $10^{12}+$ parameters
- Return prediction neural networks (Gu, Kelly, and Xiu, 2020) use 30,000+ parameters
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...But this is incorrect!

- Image/NLP models with astronomical parameterization—and *exactly fit* training data—are best performing models out-of-sample (Belkin, 2021)
- Evidently, modern machine learning has turned the principle of parsimony on its head

... And It's Happening In Finance Too

- Finance lit: Rapid advances in return prediction/portfolio choice using ML
- Large empirical gains over simple models
- Little theoretical understanding of why, and significant skepticism from old guard

What We Do: Building the "Case" for Financial ML

- **Main theoretical result**
  - Portfolio performance (Sharpe ratio) generally *increasing* in model complexity
- Explain the intuition, answer the skeptics
  - Prior evidence of empirical gains from ML are *what we should expect*
- Provide direct empirical support for theory

Problem Formulation

True Model: $R_{t+1} = f(G_t) + \epsilon_{t+1}$

- Predictors G may be known to the analyst, but the **prediction function f is unknown**
- Analyst cannot know true model, so instead she approximates f with large neural network:

$$f(G_t) \approx \sum_{i=1}^{P} S_{i,t}\beta_i$$

- Each $S_{i,t} = \tilde{f}(w_i' G_t)$ is a known nonlinear function of original predictors

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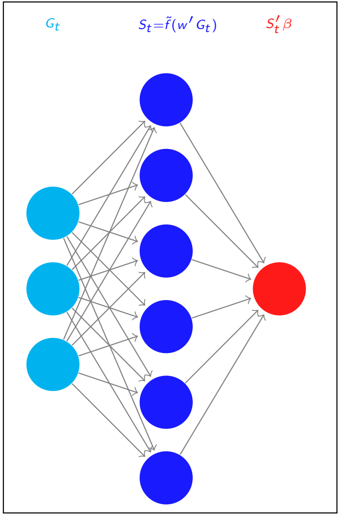

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The Choice:

- Given T data points, decide on "complexity" (number of features P) to use in approximating model

The Tradeoff:

- Simple model ($P << T$) has low variance thanks to parsimony, but is coarse approximator of f
- Complex model ($P > T$) is good approximator, but may behave poorly (and requires shrinkage)

Our Central Research Question:

- Which P should analyst opt for? Does benefit of more parameters justify their cost?

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Answer:

- Use the largest P you can compute

Environment

Model and Strategy

Model

$$R_{t+1} = S_t'\beta + \epsilon_{t+1}$$

- Single asset, R_{t+1}
- $P \times 1$ vector of predictor variables, S_t
- Linearity is without loss of generality
- Assumptions on β
  - Predictability identically distributed across signals (in expectation)
  - Total predictability is fixed

Timing Strategy

$$R^{\pi}_{t+1} = \pi_t R_{t+1}, \quad \pi_t = \beta' S_t.$$

- π_t: Timing weight scales asset position up/down to exploit time variation in expected return
- (Approximately) optimal for unconditional Sharpe maximization, convenient to analyze
- Results not sensitive to details of π function

Environment

Big Data + Big Model Limits

Goals of Theoretical Analysis

1. Characterize expected **out-of-sample** behaviors (prediction and portfolio performance)
   - All moments reported in "expected out-of-sample" form, *nothing in-sample*
2. Emphasize behavior of **machine learning** models, i.e., when number of parameters P is large
   - Differentiate between **correctly specified** versus **mis-specified** models

Tools

- Joint limits as numbers of observations and parameters are large, $T, P \to \infty$
- **Model complexity**, defined as $c = P/T$, arises as primary determinant of out-of-sample behaviors
- We leverage limiting results of random matrix theory

Why Do Big Models "Work"? Background From Least Squares

$$\boxed{R_{t+1} = \beta' S_t + \epsilon_{t+1}}$$

- Estimator when $P \leq T$: OLS

$$\hat{\beta} = \left(\frac{1}{T} \sum_t S_t S_t' \right)^{-1} \frac{1}{T} \sum_t S_t R_{t+1}$$

  - T equations in P unknowns $\Rightarrow$ Unique solution for $\hat{\beta}$

- Estimator when $P > T$: Ridge Regression

$$\hat{\beta}(z) = \left(zI + \frac{1}{T} \sum_t S_t S_t' \right)^{-1} \frac{1}{T} \sum_t S_t R_{t+1}$$

  - More unknowns (P) than equations (T) $\Rightarrow$ Multiple solutions for $\hat{\beta}$
  - "Ridgeless" regression, $\lim_{z \to 0} \hat{\beta}(z) \equiv \hat{\beta}(0^+)$. Smallest variance solution that exactly fits training data

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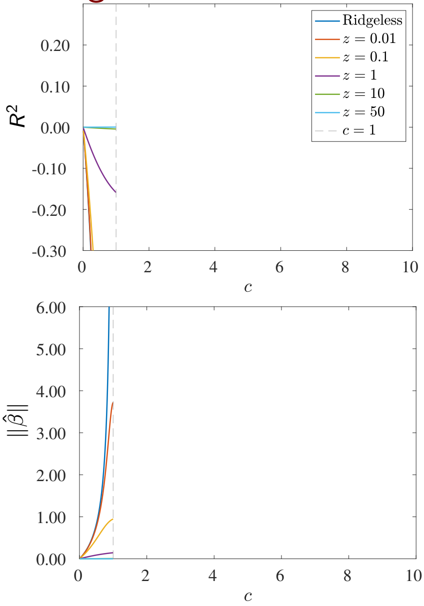

- $P, T \to \infty$ and $P/T \to c$
- $c = 0$: "Standard" asymptotics

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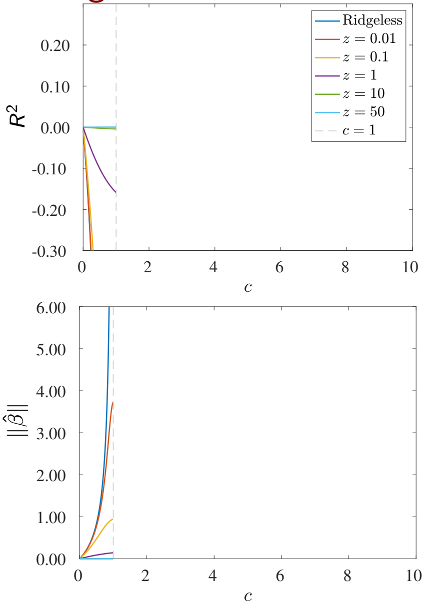

- $P, T \to \infty$ and $P/T \to c$
- $c = 0$: "Standard" asymptotics
- As $c \to 1$, expected out-of-sample R^2 tends to $-\infty$
  - Wild variance of estimates
  - Common interpretation is overfit: Exactly fit training data, but poor generalization out-of-sample
- Worrisome for trading strategy!
- Regularization helps mitigate

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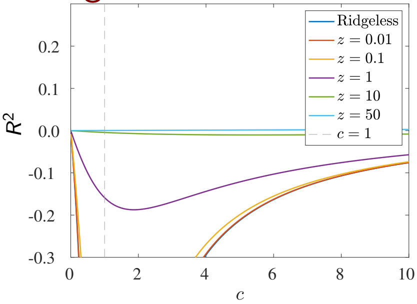

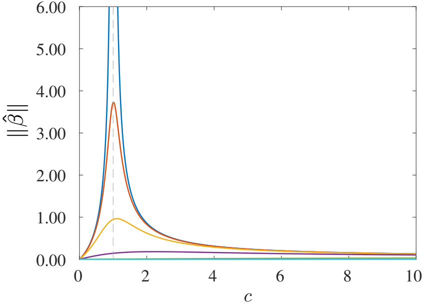

- When $c > 1$, "ridgeless" is $\lim_{z \to 0} \beta(z)$
- Counter-intuitively, OOS R^2 begins to *rise* with model complexity! Why?

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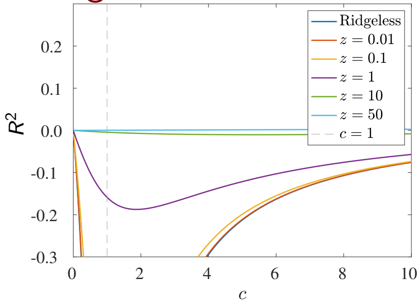

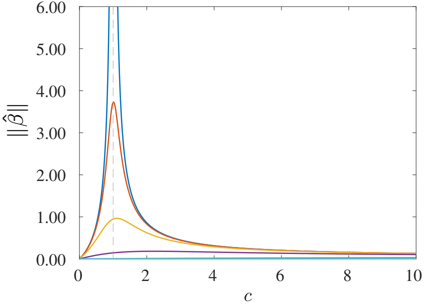

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- Many β's exactly fit training data, ridgeless selects one with smallest $\|\beta\|$
- Higher $c \Rightarrow$ more solutions to search over $\Rightarrow$ smaller $\|\beta\|$ with perfect training fit
- Shrinking β estimate despite $z \to 0 \Rightarrow$ forecast variance drops, R^2 improves

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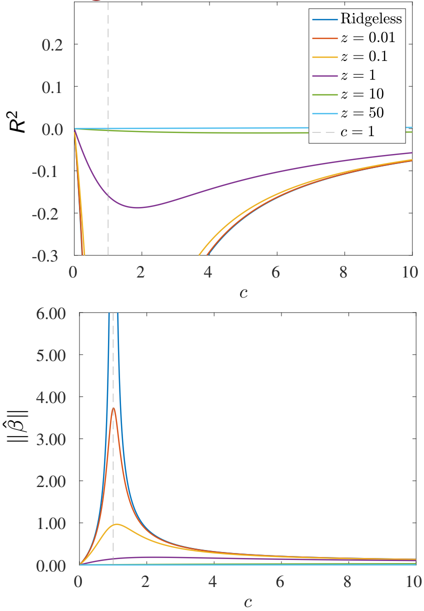

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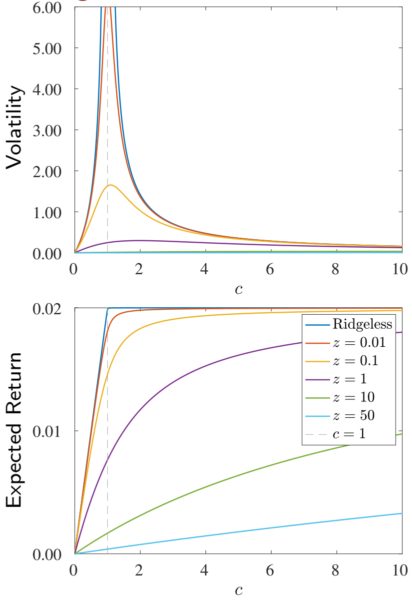
- Active topic of research in ML literature ("benign overfit," "double descent," ...)
- Challenges dogma of parsimony

Why Do Big Models "Work"? The Trading Strategy Perspective



- $c = P/T$

1. Strategy variance
   - As $c \to 1$, strategy variance blows up. One β exactly fits training data, but it has high variance
   - When $c > 1$, variance *drops* with model complexity! Why?
   - Many β's exactly fit training data, ridge selects one with small variance
2. Strategy expected returns
   - ER low for $c \approx 0$ due to poor approximation of true model
   - Raising model complexity monotonically increases ER
   - Note the contrast with R^2!

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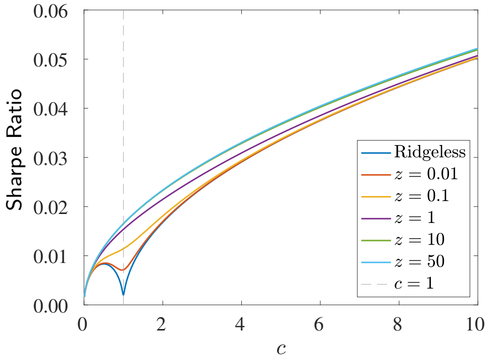

Main theory result

- Expected return always rises with model complexity (benefit of improved approximation)
- At same time, complex models have surprisingly low variance
- As a result, Sharpe ratio strictly increases with complexity

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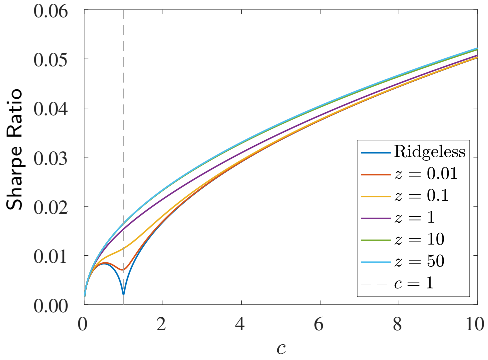

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Complexity is a virtue. Approximation benefits dominate costs of heavy parameterization

- Paper provides general, rigorous theoretical statements and proofs that underlie plots
- Plots calculated from our theorems in a reasonable calibration

Empirical Analysis

- Analyze exact empirical analogues to theoretical comparative statics
- Focus on a cornerstone of empirical finance research—forecasting aggregate market return
- To make conclusions as easy to digest as possible, study conventional setting with conventional data
    - Forecast target is monthly return of CRSP value-weighted index 1926–2020
    - Info set consists of 15 predictor variables[†] from Welch and Goyal (WG, 2008)

[†] This list includes (using mnemonics from their paper): dfy, infl, svar, de, lty, tms, tbl, dfr, dp, dy, ltr, ep, b/m, and ntis, as well as one lag of the market return.

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Random Fourier Features

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- Need framework to smoothly transition from low to high complexity
- Adopt ML method known as "random Fourier features" (RFF)
  - Let G_t be 15×1 predictors. RFF converts G_t into

$$S_{i,t} = \sin(\omega_i' G_t), \quad \omega_i \sim iidN(0, \gamma I)$$

  - $S_{i,t}$: Random lin-combo of G_t fed through non-linear activation
- For fixed inputs, can create arbitrarily large (or small) feature set
  - Low-dim model (say $P = 1$) draw a single random weight
  - High-dim model (say $P = 10{,}000$) draw many weights

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  - Low-dim model (say $P = 1$) draw a single random weight
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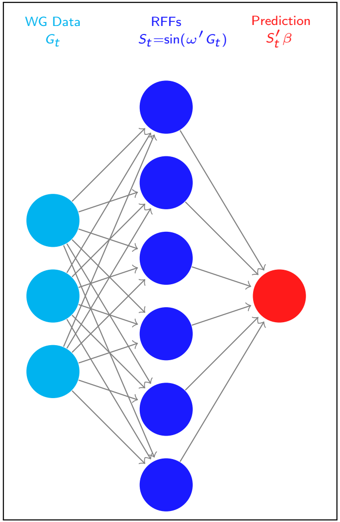
- In fact, RFF is two-layer neural network with fixed weights (ω_i) in first layer and optimized weights (regression β) in second layer



Empirical Analysis

Training and Testing

- One-year rolling training window ($T = 12$) and large set of RFFs
  i. Reach extreme levels of model complexity with smaller P and thus less computing burden
  ii. Demonstrates virtue of complexity can be enjoyed in shockingly small samples
- Draw plots with model complexity $P = 1, ..., 12{,}000$ and shrinkage of $\log_{10}(z) = -3, ..., 3$

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Empirical Procedure

i. Generate 12,000 RFFs

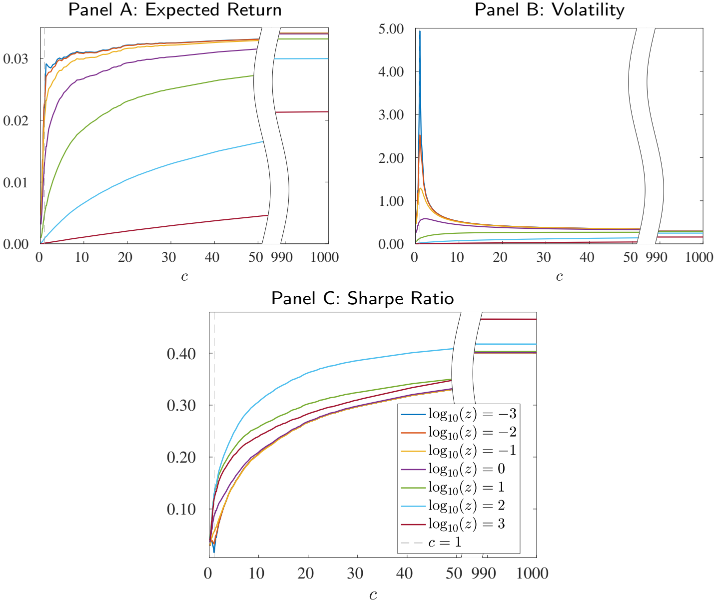
ii. Fix model defined by choice of (P, z)

iii. For each model (P, z), conduct recursive OOS prediction/timing strategy

iv. From OOS predictions, calculate ER, vol, and Sharpe of timing strategy

Out-of-sample Market Timing Performance

- Broadly: OOS behavior of ML predictions closely matches theory
- Variance explodes at $c \approx 1$ and recovers in high complexity regime
- Most importantly: OOS ER is increasing in complexity
- Sharpe of 0.4 p.a. for high complexity model. Mostly alpha/IR versus buy-and-hold with $t(\alpha) = 2.9$



Some Extensions

Virtue of Complexity Everywhere (Kelly, Malamud, and Zhou, 2022)

- Document identical pattern—OOS Sharpe ratio increasing in model complexity—in many asset classes
- US equities, international equities, bonds, commodities, currencies, and interest rates

APT or "AIPT"? The Surprising Dominance of Large Factor Models (Didisheim, Ke, Kelly, and Malamud, 2024)

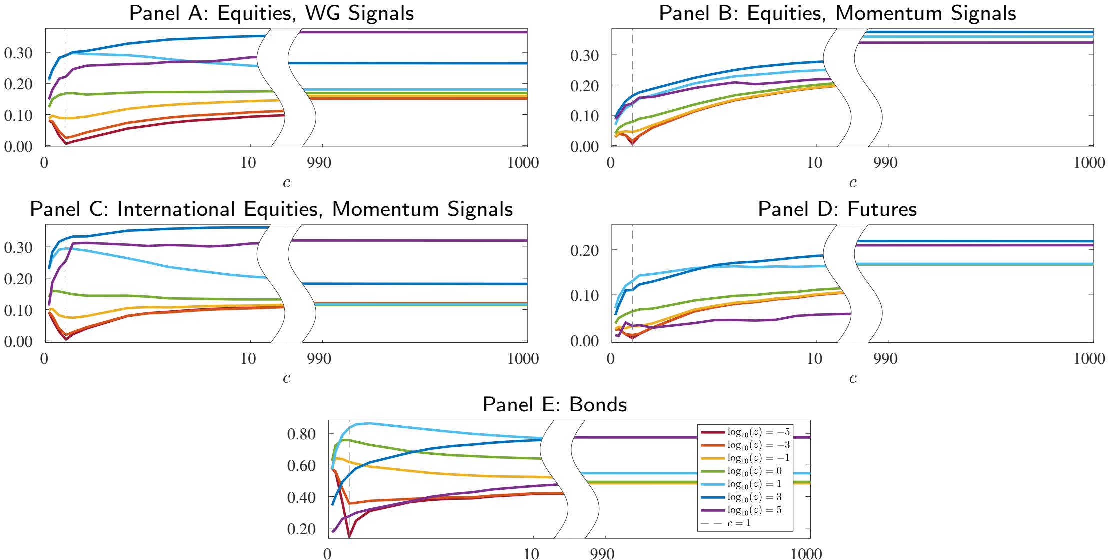
- Complexity in the cross section prediction

Artificial Intelligence Asset Pricing Models (Kelly, Kuznetsov, Malamud, Xu, 2024)

- "Deep" VoC. Embeds transformer architecture within asset pricing model

“The Virtue of Complexity Everywhere”

Kelly, Malamud, and Zhou



“APT or ‘AIPT’? The Surprising Dominance of Large Factor Models”

Kelly, Malamud, Didisheim, and Ke

- Ross’s (1976) APT conjectures small number of factors govern the joint variation of returns
- This premise + no-arbitrage arguments $\Rightarrow E[R]$ given by exposures to few common risk factors
- Most AP empirics in past fifty years occur within confines of APT—small linear factor models.

This paper

- Different conjecture: AP models with exorbitant number of factors better describe asset returns
- Rooted in theory of ML/AI: Complex’ statistical models universally outperform smaller models
- “AIPT”

Complexity in the Cross Section: A Brief History

$$E[M^{\star}_{t+1} R_{i,t+1} | X_t] = 0 \ \forall i$$

SDF representable as managed portfolio: $M^{\star}_{t+1} = 1 - \sum_{i=1}^{n} w_i(X_t) R_{i,t+1}$, s.t.

- Cross-sectional asset pricing is about $w_t = w(X_t)$
- Fundamental challenge in cross-sectional asset pricing: w must be estimated
  - This is a high-dimensional (*complex*) problem
- Standard approach: Restrict w's functional form and conditioning information
  - E.g., Fama-French: $w_{i,t} = b_0 + b_1 \text{Size}_{i,t} + b_2 \text{Value}_{i,t}$ (Brandt et al. 2007 generalize)
  - Reduces parameters, implies factor model: $M_{t+1} = 1 - b_0 MKT - b_1 SMB - b_2 HML$
  - "Shrinking the cross-section" Kozak et al. (2020) — use a few PCs of anomaly factors
  - The role of theory

Complexity in the Cross Section: Machine Learning Perspective

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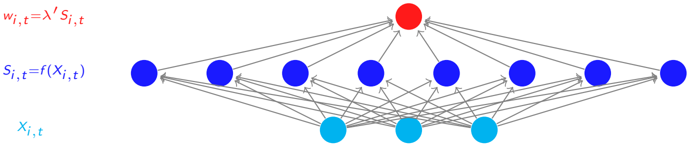

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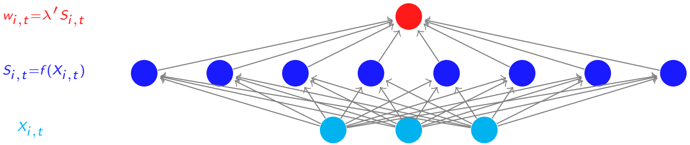

- Implies that empirical SDF is a high-dimensional factor model with factors F_{t+1} :

$$M^{\star}_{t+1} \approx M_{t+1} = 1 - \lambda' \underbrace{S_t' R_{t+1}}_{=F_{t+1} \ \in \ \mathbb{R}^{P \times 1}} = 1 - \lambda' F_{t+1} \tag{1}$$

Traditional ("Fama-French style") empirical asset pricing:

$$F_{t+1} = \sum_i \text{char}_{i,t} R_{i,t+1} \quad \text{e.g., char} \in \{\text{rank(B/M), rank(Size), rank(Prof.), rank(Inv.)}\}$$

- SDF is MVE portfolio of factors
- Evaluation:
  - Sharpe ratio of MVE
  - Pricing Error among test assets $\left(\sum_{j \in \text{test assets}} \alpha_j^2\right)$

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Complex version: Rather than plain B/M etc., build thousands of nonlinear composite signals

$$S_{k,t} = \sin(w_{k,B/M} B/M + w_{k,Size} Size + w_{k,Prof.} Prof. + w_{k,Inv.} Inv.)$$

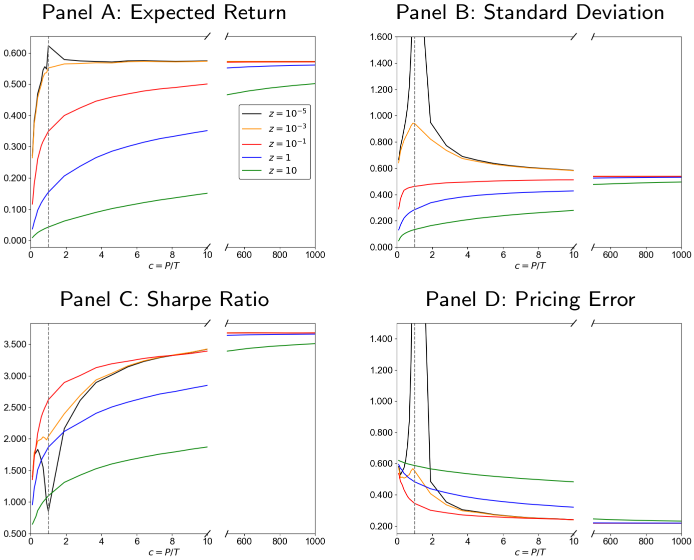
$$F_{t+1} = \sum_i \text{char}_{i,t} R_{i,t+1} \quad \text{e.g., char} \in \{\text{rank}(\text{S}_k),\ k = 1, ..., 10{,}000\}$$

- $F_{t+1} = \sum_i \text{char}_{i,t} R_{i,t+1}$
- Still, SDF is MVE portfolio of factors
- Out-of-sample evaluation is critical

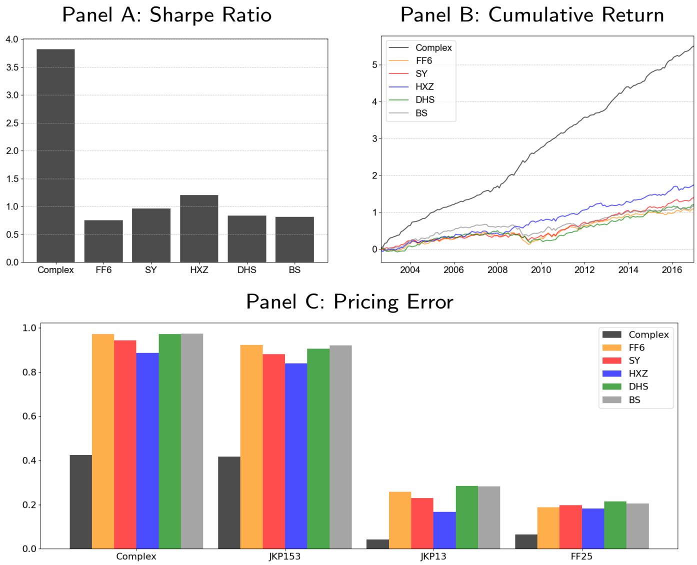
Empirical Analysis

- Study conventional setting with conventional data
  - Monthly return of US stocks from CRSP 1963–2021
  - Conditioning info ($X_{i,t}$): 130 stock characteristics from Jensen, Kelly, and Pedersen (2023)
- Out-of-sample performance metrics are:
  - SDF Sharpe ratio
  - Mean squared pricing errors (alphas of test assets)

Out-of-sample Performance of Complex Factor Models



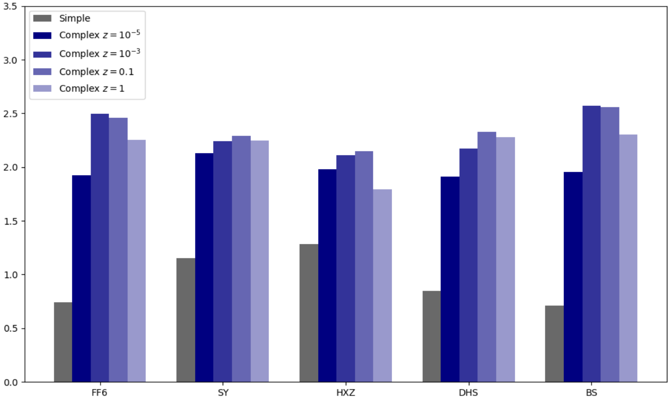
Performance Comparison of Complex and Benchmark Factor Models



*FF6: Fama and French (2015), SY: Stambaugh and Yuan (2017), HXZ: Hou et al. (2015, 2021), DHS: Daniel et al. (2020), BS: Barillas and Shanken (2018)

The Nonlinear Fama-French Model

- Restrict input data to model-specific characteristics (e.g., size, value, prof., inv., mom. for FF6)
- Generate many nonlinear basis functions (a neural network) of these 5 characteristics
- Evaluate out-of-sample Sharpe ratio:



Economics of the Complex SDF

What is responsible for mean-variance efficiency of complex SDF?

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At statistical level: Complex SDF more successfully optimizes the Sharpe ratio objective

Economics of the Complex SDF

What is responsible for mean-variance efficiency of complex SDF?

At statistical level: Complex SDF more successfully optimizes the Sharpe ratio objective

At economic level:

- SDF $\leftrightarrow$ IMRS, shaped by investors' info/expectations about future macroeconomic conditions
- MVE portfolio is projection of the IMRS onto the space of traded assets
- More efficient tradable SDF $\leftrightarrow$ better honing in on expectations about future economy
- Fama (1991) study of market returns and the macroeconomy:

  "the general hypothesis ... is that information about the production of a given period is spread across preceding periods and so affects the stock returns of preceding periods."

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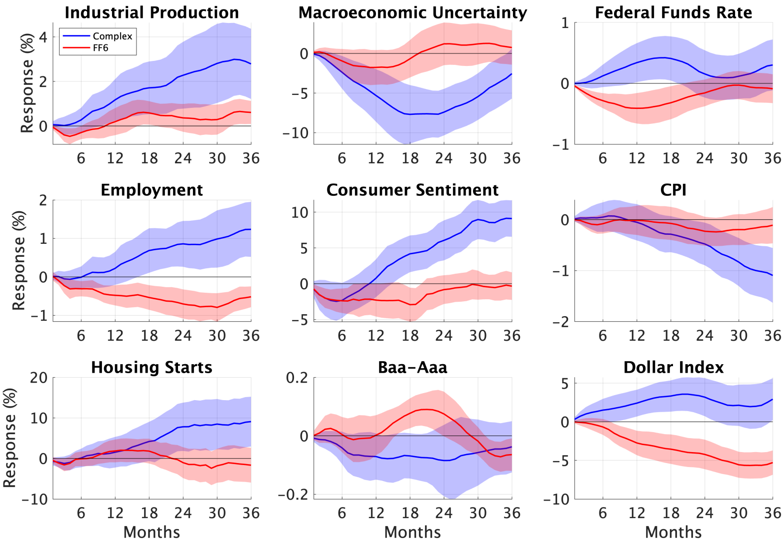
- Approach: Impulse response functions using the method of local projections (Jorda, 2005)

$$y_{t+h} - y_t = a_h + b_{SDF,h} \left(\sum_{j=0}^{11} R^M_{t-j} \right) / \sigma^M + \sum_{j=0}^{2} b_{j,h} y_{t-j} + e_{t+h}, \quad h = 1, ..., 36.$$

- y_t contains measures of macro activity in monthly log levels (ind. prod. macro uncertainty, Fed Funds rate, empl., cons. sentiment, CPI, housing starts, credit spread, oil prices, exchange rate index.

Economics of the Complex SDF



1σ rise in annual SDF returns predicts...

- 3% rise in IP over three years
- 7.5% drop in macro uncertainty over two years
- Large and significant rise in employment (1.25%), sentiment (9%), housing (9%), and dollar (2.5%) over three years
- Fall in inflation and credit spread

Conclusions, I

- Asset pricing and asset management in midst of boom in ML research
- We provide new, rigorous theoretical insight into the behavior of ML models/portfolios
- Contrary to conventional wisdom: Higher complexity improves model performance

Virtue of Complexity: Performance of ML portfolios can be improved by pushing model parameterization far beyond the number of training observations

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- *Not* license to add arbitrary predictors to model. Instead, we recommend
  i. including all plausibly relevant predictors
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  i. including all plausibly relevant predictors
  ii. using rich non-linear models rather than simple linear specifications
  - Doing so confers prediction/portfolio benefits, even when training data is scarce and particularly when accompanied by shrinkage
- In canonical empirical problems we find
  - OOS performance roughly doubles relative status quo models

Conclusions, II

- Clashes with philosophy of parsimony frequently espoused by economists
- Two oft-repeated quotes from famed statistician George Box:

All models are wrong, but some are useful.

Since all models are wrong the scientist cannot obtain a 'correct' one by excessive elaboration. On the contrary, following William of Occam, he should seek an economical description of natural phenomena. Just as the ability to devise simple but evocative models is the signature of the great scientist so overelaboration and overparameterization is often the mark of mediocrity.

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Occam's Blunder? Small model is preferable only if it is correctly specified. But models are never correctly specified. Logical conclusion?

Appendix Slides

SDF Sharpe Ratio By Market Capitalization

“VoC” is not about limits to arbitrage