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# Factor Investing in Fixed-Income – Cross-Sectional and Time-Series Momentum in Sovereign Bond Markets

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# Foreword

This paper has been produced as part of the "ETF, Indexing and Smart Beta Investment Strategies" Research Chair at EDHEC-Risk Institute, in partnership with Amundi.

Looking at momentum in fixed-income markets at the security level is very important, because studies that employ 'synthetic' zero-coupon bonds can be vitiated by the well-known serial autocorrelation of pricing errors, which can masquerade as a momentum effect. To our knowledge, no empirical study of momentum in Treasuries has looked at the problem at this level of granularity.

In this paper, "Factor Investing in Fixed-Income – Cross-Sectional and Time-Series Momentum in Sovereign Bond Markets", we undertake a systematic, security-level analysis of momentum and reversal strategies in US Treasuries covering more than 40 years of data.

We distinguish between what we call 'market' and 'self' time-series momentum (reversal) strategies, and present an exact identity between these two time-series and the cross-sectional momentum (reversal) strategies. This identity helps us identify the sources of profitability of the various strategies, and raises interesting question regarding the contribution of the first and second principal components of yield changes.

We find that there exist look-back and investment periods for which momentum times series strategies (both 'self' and 'market') give rise to statistically and economically significant positive Sharpe ratios. We also find that, after adjusting for duration, the reversal cross-sectional strategy has even larger Sharpe ratios, and is profitable over a wider range of look-back and investment periods. We argue that the explanation for this finding is related to the mean reverting properties of the yield-curve slope.

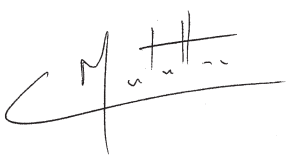
Finally, we show that the duration-adjusted reversal cross-sectional strategy can be successfully implemented in a long-only fashion.

In a companion paper, we propose a definition of value in Treasury bonds which, we believe, is more satisfactory than definitions found in the recent literature, and that allows for statistically significant and economically relevant predictions of cross-sectional excess returns.

I would like to thank Riccardo Rebonato and Jean-Michel Maeso for their leadership in this research effort, and Laurent Ringelstein and Dami Coker for their efforts in producing the final publication.

I would also like to extend particular thanks to Amundi for their support of this research chair.

We wish you a useful and informative read.



Lionel Martellini
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Lionel Martellini is Professor of Finance at EDHEC Business School and Director of EDHEC-Risk Institute. He has graduate degrees in economics, statistics, and mathematics, as well as a PhD in finance from the University of California at Berkeley. Lionel is a member of the editorial board of the *Journal of Portfolio Management* and the *Journal of Alternative Investments*. An expert in quantitative asset management and derivatives valuation, his work has been widely published in academic and practitioner journals and he has co-authored textbooks on alternative investment strategies and fixed-income securities.

**Riccardo Rebonato** was previously Global Head of Rates and FX Research at PIMCO. He also served as Head of Front Office Risk Management and Head of Clients Analytics, Global Head of Market Risk and Global Head of Quantitative Research at Royal Bank of Scotland (RBS). Prior joining RBS, he was Head of Complex IR Derivatives Trading and Head of Head of Derivatives Research at Barclays Capital. Riccardo Rebonato has served on the Board of ISDA (2002-2011), and has been on the Board of GARP since 2001. He was a visiting lecturer in Mathematical Finance at Oxford University (2001-2015). He is the author of several books, in particular having published extensively on interest rate modelling, risk management, and most notably books on SABR/LIBOR Market Model pricing of interest rate derivatives, as well as on the use of Bayesian nets for stress testing and asset allocation. He has published articles in international academic journals such as *Quantitative Finance*, the *Journal of Derivatives* and the *Journal of Investment Management*, and has made frequent presentations at academic and practitioner conferences.

# 1. Introduction and Motivation

# 1. Introduction and Motivation

This study presents a systematic empirical investigation of the protability of momentum and reversal strategies in US Treasuries, using more than 40 years of daily data at the individual-security level. Looking at momentum at the security level is very important, because studies that employ `synthetic' zero-coupon bonds can be vitiated by the well-known serial autocorrelation of pricing errors, that can masquerade as a momentum effect.[1] To our knowledge, no empirical study of momentum in Treasuries has looked at the problem at this level of granularity. We also extend and generalise the known relationships among time-series and cross-sectional momentum strategies, and, by doing do, establish exact identities that are of great help in interpreting the empirical results.

Momentum strategies have been found to be profitable in a wide number of asset classes. One of the best-known early academic documentations of the protability of a momentum strategy is Jegadeesh and Titman (1993), who document, over the period 1965-1989, a statistically significant positive performance for dollar-neutral cross-sectional momentum strategies that purchase best performing US stocks over the past 3 to 12 months, sell the losers and hold the position for 3 to 12 months. Cross-sectional momentum strategies have also been studied in the US equity market by Moskowitz and Grinblatt (1999), and in European stock markets by Rouwenhorst (1998), Gupta et al. (2010)). Similar cross-sectional strategies have then been found profitable in currencies (see, eg, Menkho et al. (2012)). All these cross-sectional strategies consist of buying securities that recently outperformed their peers over the past 3 to 12 months and selling those that recently underperformed their peers over the past 3 to 12 months.

One can also define time-series momentum, namely the strategy of looking at the past performance of each security over the last 3 to 12 months, and of buying (selling) those with positive(negative) past performance over a certain investment period. When looking at time series momentum, Moskowitz et al. (2012) report, for a 12-month time series momentum strategy with a 1-month holding period, a significant time-series momentum protability for a vast number of asset classes such as equity indices, currency, commodity and bond futures. So pervasive is the protability of the time-series strategy that Assness, Moskowitz and Pedersen (2013) claim to find "(value and) momentum everywhere", implying that the 12-month past returns of almost any security or currency is a positive predictor of its future return.

The results from these studies paint a compelling but complex picture, because returns from time-series and cross-sectional momentum are well-known to be related. See, eg, Moskowitz, Ooi and Pedersen (2012), who present a "simple, formal" decomposition of the expected returns from cross-sectional and time-series momentum in terms of cross-sectional variances and auto- and cross-covariances. Their decomposition builds on early work by Lo and Mackay (1990), who looked at reversals, and by Lewellen (2002). All these studies clearly show that the expected return from these momentum strategies can be expressed in terms of auto serial covariance, cross-serial covariance and cross-sectional variance of returns. However, disentangling the various

1 - For instance, the widely used zero-coupon bond prices by Gurkaynak, Sack and Wright (2007) are obtained by fitting the Nelson-Siegel (1987) model to the market prices of coupon-bearing bonds. As the authors recognise, these fitted prices suffer from serially correlated pricing errors.

# 1. Introduction and Motivation

contributions, and making economic sense of the different sources of protability for identical lags, is less than straightforward, especially in the case of fixed-income securities.

We show, in this paper, that by introducing a third momentum strategy (defined in what follows), the relationships between the three momentum strategies can be turned into an exact identity. This identity can be very helpful in understanding the economic relationships between the protability of different momentum and, as we shall discuss, it becomes particularly illuminating in the case of US Treasuries, which are the focus of this study.

> By considering these three strategies together, we show that a universal exact identity relationship...

The third momentum strategy alluded to above is what we call the time-series-market momentum strategy (to distinguish it from the customary time-series momentum strategy described earlier, which we dub "time-series-self mo- mentum" strategy). In the time-series-market momentum strategy, the investor chooses whether to go long or short a given security not based on the past performance of that security, but of the market as a whole — where in our context by "market" we understand the universe of securities to which the security in question naturally belongs, such as the universe of Treasury bonds in the case under study.

By considering these three strategies together, we show that a universal exact identity relationship must exist among the returns from the cross-sectional momentum, the time-series-self momentum and the time-series-market momentum strategy. Indeed, we prove in Appendix I that the expected returns from the three strategies must be linked by the following relationship:

$$E\left[\pi_t^{xs,r}(k)\right] = E\left[\pi_t^{ts,r1}(k)\right] - E\left[\pi_t^{ts,r2}(k)\right] \quad (1)$$

In Equation (1) *k* indicates the lag, the superscripts, *ts*, *r*1 and *ts*, *r*2, identify the time-series self an the time-series market strategy for reversals, respectively, *xs*, *r* signifies the cross-sectional reversal strategy, $\pi(t)$ denotes the return at time *t* from investing for *k* months, and *t* indexes the time. (The letters *r* and *m* are used to distinguish reversals from momentum strategies.)[2]

This equation shows that, for each lag, the protability of the cross-sectional strategy is given by the difference in protability of the two time series strategies. So, for instance, if the expected returns for cross-sectional reversals are negative, then the expected return for the self-time-series momentum must be larger than the market-time-series momentum strategy. Or, if the difference in the two time series strategies is negative for, say, reversals, then the cross-sectional reversal strategy must be negative. This means that the cross-sectional momentum strategy must be positive (because for each strategy, time-series or cross-sectional, momentum and reversal profits are equal in magnitude and opposite in sign).

# 1. Introduction

## 1.1 The Intuition

Why are these relationships so interesting in the case of momentum and reversals in US Treasuries? Consider the time-series-market momentum strategy. Since the time-series strategy requires the investor to take same-sign positions in all the securities in the universe, this means that, depending on the sign of past returns, the strategy requires the investor to take a long or short duration position. So, the protability of this strategy is largely determined by the future behaviour of the first ("level") principal component. When, instead, we look at cross-sectional momentum strategies, we take long and short positions in different Treasury bonds, and we are considering the relative performance of bonds of different maturity. Since, again, change in yields are dominated by the changes in the first principal component, almost invariably the relative winners and losers are to be found at opposite ends of the maturity spectrum.[3] This means that the winners versus losers position put in place in a cross-sectional momentum strategy result, in essence, in a "steepener/attener" strategy, which is exposed to first order to changes in the second principal component (the "slope").

At first blush, the two strategies therefore seem to have very little in common, and their protability appears to be driven by changes in quantities (the first two principal components), which are, by construction, designed to be at least uncorrelated, if not exactly independent. Yet, as Equation (1) shows, the sum of their protabilities must equal exactly the protability of the time-series-self momentum strategy. If we consider this strategy more closely, the very high degree of correlation between bond yields (which is of course related to the predominant importance of the first principal component) would suggest that, most of the time, it should be very similar to the time-series momentum-market strategy. Yet, the differences between the two time-series strategies must be large enough to be equal to the protability of the cross-sectional strategy – which, as we show in Section (4), is very significantly profitable.

## 1.2 Our Main Results

We discuss at greater length these findings in Section (4), but for the moment we highlight the following results:

1. We find that the time-series self momentum strategy is very significantly profitable for look-back and holding periods of 9 and 12 months.
2. We find that the time-series market momentum strategy is very significantly profitable for look-back and holding periods of 9 and 12 months.
3. We find that the cross-sectional momentum strategy is not statistically significantly profitable at any length. However, after duration adjustment of the notional, the cross-sectional *reversal* strategy is statistically very significantly profitable for look-back and holding periods of 6, 9 and 12 months.
4. We find that we can implement a long-only strategy based on duration-adjusted reversals and momentum that significantly outperform the market portfolio.
5. We document that it is indeed important to carry out momentum studies at the individual security level, rather than using synthetic discount bonds.

In the rest of the paper, we described the data we have used, we explain how these results have been obtained, and we discuss the results.

3 - If the yield curve moves (roughly) in parallel, if rates move up, the short-maturity end of the yield shows the lowest losses (is the relative winner), and if rates go down long-maturity rates are the winners.

# 2. The Data

# 2. The Data

The data used for the study are the daily close of-business-day prices of 1,562 US Treasury coupon bonds over the period 27 December 1973 to 29 June 2018.[4] We excluded from the data-set prices of individual bonds that were deemed to be erroneous. The exclusion was determined by setting a threshold in standard deviations for the price changes, and then excluding bonds with price moves that exceeded the threshold *while the other bonds in the universe for that day did not show a similar move*. In addition, at each date *t*, we only considered bonds with a time-to-maturity higher than or equal to 2 years and smaller or equal to 15 years. Finally, we computed bond total return price series by assuming each coupon paid by a given bond to be reinvested in the same bond. Table 1 reports the summary statistics of the sovereign bond sample. It contains 46,578 monthly return observations.

*Table 1: Descriptive statistics of the bond data used*

| Sample period | Dec 1973 - June 2018 |
|---|---|
| Bond observations | 46,578 |
| Monthly returns - mean (perc) | 0.52 |
| Monthly returns - median (perc) | 0.38 |
| Monthly returns - first quantile (perc) | -0.26 |
| Monthly returns - third quantile (perc) | 1.22 |
| Duration - mean (years) | 4.32 |
| Duration - median (years) | 3.92 |
| Duration - first quantile (years) | 2.77 |
| Duration - third quantile (years) | 5.58 |
| Time to maturity - mean (years) | 5.45 |
| Time to maturity - median (years) | 4.54 |
| Time to maturity - first quantile (years) | 3.09 |
| Time to maturity - third quantile (years) | 6.92 |

4 - All these bonds are non-callable, non-puttable and non-inflation-linked. We thank ICE for providing us with the data-set used for our empirical analysis.

# 3. Construction of the Strategies

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In order not to break the flow of the presentation, in Appendix I we present the precise methodology used to construct the cross-sectional, time-series-self and time-series-market strategies. At this stage we simply point out that we look at:

1. Cross-sectional Momentum: long-short strategies (with and without duration adjustment), and long-only strategies;
2. Time Series Momentum: self long-short strategies
3. Time Series Momentum: market long-short strategies

As explained in Appendix I, in all cases we deal with zero-cost portfolios. In list item 1 we mention duration adjustment. We discuss also this feature in Appendix I, but at this stage we simply mention that duration-adjustment is achieved by dividing the returns of each bond by their duration. So, the duration-adjusted market return is the cross-sectional average of the duration-adjusted bond returns. We stress that duration adjustment *does not imply duration neutralisation*, and it used to achieve an approximate risk parity (volatility parity) among the various constituent bonds.

Finally, we remark that, as commonly observed in the literature, choosing different look-back and holding periods gives different average returns, standard deviations of returns, and Sharpe ratios. The resulting high number of permutations can give rise to involuntary data mining. In his recent study, Durham (2015) tries to circumvent this problem by averaging the results over the holding and look-back periods in his study. While at first blush reasonable, we do not think that this procedure is advisable: as Lewellen (2002) points out, momentum and reversal are due to positive and negative serial autocorrelation of returns. As the same author also points out, positive autocorrelation of returns (associated with momentum) cannot continue forever, or prices would stray indefinitely from fundamentals, and at some point mean-reversion (*negative* autocorrelation) must kick in. Momentum / reversal studies test the joint hypotheses that i) this over-reaction/correction pattern does exist and ii) that there exist relatively stable (and hence predictable) characteristic lengths for the overreactions and corrections. As a consequence, it is perfectly reasonable to expect that a, say, momentum strategy may be profitable over some look-back / holding periods, and unprofitable over different periods. Averaging over periods therefore washes away variations that are partly due to noise, but partly (and importantly) due to the intrinsic periodicities (if present) of overreactions and corrections. For this reason we do not employ averaging, but report our results as function of the look-back and holding periods. As mentioned above, we try to limit the risk of data mining but constraining the look-back periods to be equal to the holding periods.

# 4. Results

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In this section, we report the results of the different momentum strategies discussed in the methodology session. More precisely, we divide the presentation between

1. strategies with notionals proportional to the strength of the signal (as described in Appendix II, and which we call "variable-notional") – see Section 4.1;
2. strategies with normalised notionals (which we define "normalised") – see Section 4.2;
3. duration-adjusted strategies – see Section 4.3;
4. long-only strategies – see Section 4.4.

## 4.1 Variable-Notional Strategies

We start by reporting in Tables (2), (3) and (4) the descriptive statistics for the time-series-self, time-series-market and cross-sectional momentum strategies. The notionals of the long and short positions were set to be proportional to the strength of the signal, as described in Appendix II. The corresponding reversal strategies simply have returns and Sharpe ratios with the opposite sign.

The first observation from examining the realised returns is the similarity between the self and market time-series momentum strategies. This is to be expected, as discussed in the Introduction, because of the high correlation among bonds, and the level- (duration-) nature of the time-series momentum strategy. We note that for 9-month look-back and holding periods the Sharpe ratio is highly significantly different from zero for both time-series strategies. As the identity in Equation (1) shows, the cross-sectional returns must be correspondingly small. Again, we can readily understand this observation if we note that the securities in the universe are highly correlated, and relative-value strategies (long the relative winners and short the relative losers) do not have much scope for profitability. This, however,

*Table 2: Descriptive statistics of the time-series-self momentum strategy*

| Time-Series-Self (momentum) | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) | -0.0002 | 0.0005 | 0.0009 | 0.0006 |
| Standard deviation (annualised) | 0.0035 | 0.0030 | 0.0039 | 0.0054 |
| Sharpe ratio | -0.0460 | 0.1723 | 0.2316 | 0.1031 |
| t-test | -0.52 | 2.75 | 4.48 | 2.30 |
| t-test (Newey-West) | -0.38 | 1.67 | 2.65 | 1.28 |

*Table 3: Descriptive statistics of the time-series-market momentum strategy*

| Time-Series-Market (momentum) | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) | -0.0001 | 0.0005 | 0.0009 | 0.0006 |
| Standard deviation (annualised) | 0.0032 | 0.0028 | 0.0036 | 0.0050 |
| Sharpe ratio | -0.0417 | 0.1829 | 0.2485 | 0.1193 |
| t-test | -0.47 | 2.92 | 4.80 | 2.66 |
| t-test (Newey-West) | -0.34 | 1.76 | 2.84 | 1.48 |

# 4. Results

*Table 4: Descriptive statistics of the cross-sectional momentum strategy. The mean returns and standard deviations have been multiplied by 100.*

| Cross-sectional (momentum) | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) [*100] | -0.0024 | 0.0009 | 0.0001 | -0.0041 |
| Standard deviation (annualised) [*100] | 0.026 | 0.028 | 0.033 | 0.043 |
| Sharpe ratio | -0.090 | 0.031 | 0.0039 | -0.095 |
| t-test | -1.02 | 0.50 | 0.08 | -2.11 |
| t-test (Newey-West) | -0.79 | 0.33 | 0.05 | -1.20 |

*Table 5: Descriptive statistics of the time-series-self normalised momentum strategy*

| Time-series-self normalised (momentum) | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) | 0.010 | 0.011 | 0.015 | 0.011 |
| Standard deviation (annualised) | 0.056 | 0.055 | 0.057 | 0.060 |
| Sharpe ratio | 0.177 | 0.200 | 0.262 | 0.190 |
| t-test | 2.01 | 3.19 | 5.09 | 4.24 |
| t-test (Newey-West) | 1.64 | 2.10 | 3.10 | 2.36 |

*Table 6: Descriptive statistics of the time-series-market normalised momentum strategy*

| Time-series-market normalised (momentum) | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) | 0.009 | 0.013 | 0.016 | 0.011 |
| Standard deviation (annualised) | 0.059 | 0.059 | 0.060 | 0.061 |
| Sharpe ratio | 0.155 | 0.213 | 0.264 | 0.0183 |
| t-test | 1.76 | 3.40 | 5.14 | 4.07 |
| t-test (Newey-West) | 1.42 | 2.23 | 3.21 | 2.30 |

*Table 7: Descriptive statistics of the cross-sectional normalised momentum strategy*

| Cross-sectional normalised (momentum) | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) | -0.0001 | 0.0014 | 0.0016 | -0.0005 |
| Standard deviation (annualised) | 0.0415 | 0.0414 | 0.0421 | 0.0433 |
| Sharpe ratio | -0.0023 | 0.0346 | 0.0392 | -0.0118 |
| t-test | -0.0263 | 0.5519 | 0.7573 | -02639 |
| t-test (Newey-West) | -0.0207 | 0.3653 | 0.4632 | -0.1485 |

does not imply that the Sharpe ratio should also be small. For variable-notional cross-sectional strategies this happens to be the case, but we show in Section 4.3 that a simple modification of how the notional is determined can turn the cross-sectional reversal strategy into the strategy with the highest and most significant Sharpe ratio across the widest range of look-back / investment periods.

# 4. Results

## 4.2 Normalised-Notional Strategies

To what extent is the strength of the signal (rather than just its sign) informative? To answer this question in this sub-section we consider results for two variations on the variable-notional strategies: for the time-series momentum-self strategy instead of having a notional depending of the relative cross-sectional performance, we set it equal to 1/*N*, where *N* is the number of bonds in the long and short portfolios. As for the time-series-market strategy, if the market return over the look-back period is positive (negative) we buy (sell) all the bonds with notional 1/*N*, and we fund with cash (rolled Treasury Bills). In Tabs (5) and (6) ) we refer to these strategies as "normalised".

As Tabs 5 to 7 show (see in particular the Sharpe ratios and the t-tests), little information is conveyed by the size of the signal, once its sign is taken not account: the time series strategies remain profitable (if anything, the equal-weight strategy extends the profitability of time series profitability to the 3-month horizon), and the cross-sectional strategy remains unprofitable (and the t-statistics point to non-significant results). As we shall see in the next section, the picture changes radically for cross-sectional strategies when we introduce duration adjustment.

## 4.3 Cross-Sectional Duration-Adjusted Strategies

As the reader will recall, the duration-adjustment is achieved by dividing the returns from each bond by its duration at the stage of the determination of the notional, as described in Section 4.3. The duration-adjusted market return is the cross-sectional average of the duration-adjusted bond returns. The attending descriptive statistics for the cross-sectional duration-adjusted strategy are shown in Tab (8).

> As mentioned, duration adjustment has a very strong effect...

As mentioned, duration adjustment has a very strong effect: after adjusting for duration (ie, roughly normalising with respect to volatility), we find not only the most highly significant protability, but also the widest range of significant look-back / investment periods (6, 9 and 12 months). In addition, the Sharpe ratios are higher than the Sharpe ratios recorded for any of the time-series momentum strategies. So, duration adjustment matters a lot. We explain the origin of this protability in Section 5.

*Table 8: Descriptive statistics of the duration-adjusted cross-sectional momentum strategy*

| Cross-sectional (reversals), duration-adjusted | | | | |
|---|---|---|---|---|
| Look-back and holding periods (months) | 3m | 6m | 9m | 12m |
| Mean return (annualised) | 0.0052 | 0.0087 | 0.0112 | 0.0128 |
| Standard deviation (annualised) | 0.0378 | 0.0376 | 0.0385 | 0.0381 |
| Sharpe ratio | 0.1372 | 0.2326 | 0.2899 | 0.3361 |
| t-test | 1.56 | 3.71 | 5.60 | 7.50 |
| t-test (Newey-West) | 1.16 | 2.21 | 3.13 | 4.11 |

# 4. Results

## 4.4 Long-Only Strategies

From the perspective of important classes of institutional investors with investment constraints, long-only strategies are particularly relevant. We therefore present, in this section, the results for three long-only strategies:

1. the strategy of giving equal weights to all the bonds;
2. the strategy of giving duration-adjusted positive weights to the past losers; and
3. the strategy of giving duration-adjusted positive weights to the past winners.

The results are shown in Fig (1) and in tabular form in Tab (9).

We discuss the significance of the results in the next section, when we put it the context of the protability of the various strategies.

# 5. Discussion of the Results

# 5. Discussion of the Results

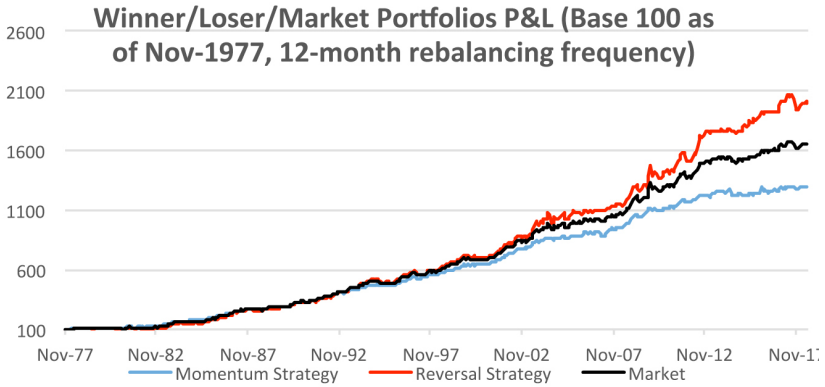
The first observation from the results above is that, with variable notionals as described in the Appendix, both self and market time-series momentum strategies are profitable at the 99% confidence level for look-back and investment periods of 9 months. In the case of Treasury bonds, for all lags the market and self time series strategies give very similar results. This is to be expected, because of the high-correlation among securities. Due to Equation (1), this implies that the profits from the variable-notional cross-sectional strategy will be small. Again, in the case of US Treasuries this is to be expected, because the high correlation among securities gives little scope for unleveraged returns based on being long some securities and short others. In itself, the smallness of the cross-sectional returns with variable notionals does not make the strategy unattractive, because, within limits, the returns can be amplied by leverage. However, as Tabs (4) and (7) show, the associated Sharpe ratios with variable notionals are statistically indistinguishable from zero. The picture for cross-sectional strategies changes radically if we adjust the variable notionals by duration. Now the cross-sectional strategy becomes even more statistically significant than the time-series strategies, not only for the 9- and 12-month investment horizons, but also for the 6-month one. See Tab (8).

Why does duration adjustment bring about such a marked improvement in the cross-sectional strategy? To understand the origin of this

*Table 9: Descriptive statistics of the long-only duration-adjusted cross-sectional strategy*

| Long-only cross-sectional, duration-adj. | | | |
|---|---|---|---|
| Statistic | Winners' Portfolio | Losers' Portfolio | Market |
| Mean return (annualised) | 0.064 | 0.077 | 0.071 |
| Mean return (annualised) | 0.044 | 0.076 | 0.059 |
| Sharpe ratio | 0.43 | 0.42 | 0.44 |
| t-test (mean TR = null) | 6.45 | 9.344 | 7.61 |
| t-test (Difference versus market) | -1.92 | 1.76 | - |

*Figure 1: The cumulative returns from giving equal weights to all bonds (curve labelled `Market'); duration-adjusted weights to past winners (curve labelled `Winner Portfolio'); and duration-adjusted weights to past losers (curve labelled `Loser Portfolio')*



# 5. Discussion of the Results

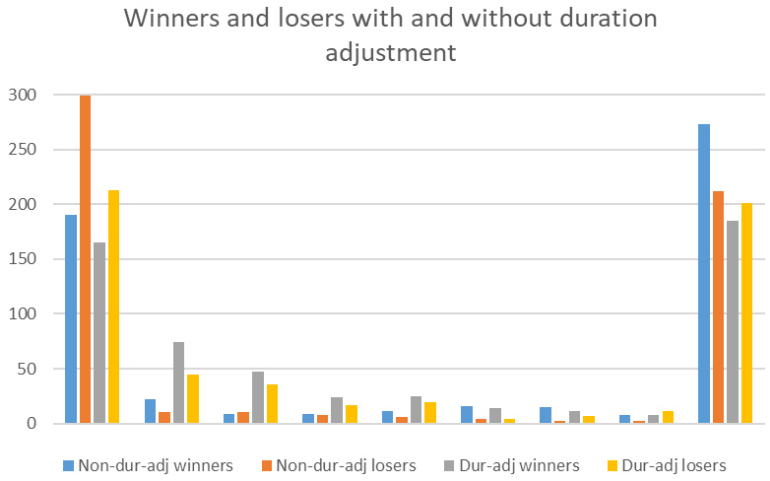
improvement in performance, recall first that, by duration adjusting, in order to establish winners and losers we divide both the market and the security returns by their durations. Since yield curve moves are dominated by quasi-parallel shifts in yields,without duration adjustment winners and losers tend to be found at either end of the maturity spectrum (the long end if rates have fallen, and the short end if they have risen). If the Treasury returns were only due to parallel moves in the yield curves, dividing by the duration would approximately equalise the returns from bonds of different maturities, and there would be no reason to find winners and losers preferentially at either end of the maturity spectrum. Indeed, we do find that, after duration adjustment, the polarisation of winners and losers at either end of the yield curve is less pronounced, and intermediate maturity bonds are now often picked as winners. See Fig 2. However, there still remains a strong predominance of long- or short-maturity bonds among the winners and losers. Since we have neutralised against parallel movements in the yield curve, this means that the profitability of the duration-adjusted strategy is closely linked to changes in the slope of the yield curve.

It is well known, in turn, that the yield curve slope (closely related to the second principal component of yield changes) is strongly mean-reverting. See, eg, Diebold and Rudebusch (2013). The success of the cross-sectional duration-adjusted *reversion* strategy therefore appears to be linked to the mean-reverting properties of the yield curve slope. More precisely, by determining the winners and losers after dividing by duration, weights

*Table 10: Frequency of winners and losers as a function of maturity (from 2 to 10 years on the x axis) with and without duration adjustment*

| Maturity (years) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| Non-dur-adj winners | 190 | 22 | 9 | 9 | 11 | 16 | 15 | 8 | 273 |
| Non-dur-adj losers | 299 | 10 | 10 | 8 | 6 | 4 | 2 | 2 | 212 |
| Dur-adj winners | 165 | 74 | 47 | 24 | 25 | 14 | 11 | 8 | 185 |
| Dur-adj losers | 213 | 45 | 36 | 17 | 19 | 4 | 7 | 11 | 201 |

*Figure 2: The relative winners and losers with and without duration adjustment*



# 5. Discussion of the Results

are created that can exploit the mean-reverting properties of the slope, and, indeed, we find that a cross-sectional *reversion* strategy becomes very profitable.

The effect of duration adjustment on the composition of winners and losers is shown graphically in Fig (2) and in tabular form in Tab (10). Note how the frequency of the short-maturity winners (losers) decreases from 190 (299) to 165 (213) and the frequency of long-maturity winners (losers) decreases from 273 (212) to 185 (201). Yet, even after duration adjustment, the ratio of the losers (winners) in the 10-year maturity bucket to the sum of the losers (winners) in the maturity 6- , 7- , 8- and 9-year maturity buckets remains 4.90 (3.19); and the ratio of losers (winners) in the 2-year maturity bucket to the sum of the losers (winners) in the maturity 3- , 4- and 5-year maturity buckets remains 1.68 (1.13).

# 6. The Importance of Security-Level Analysis

# 6. The Importance of Security-Level Analysis

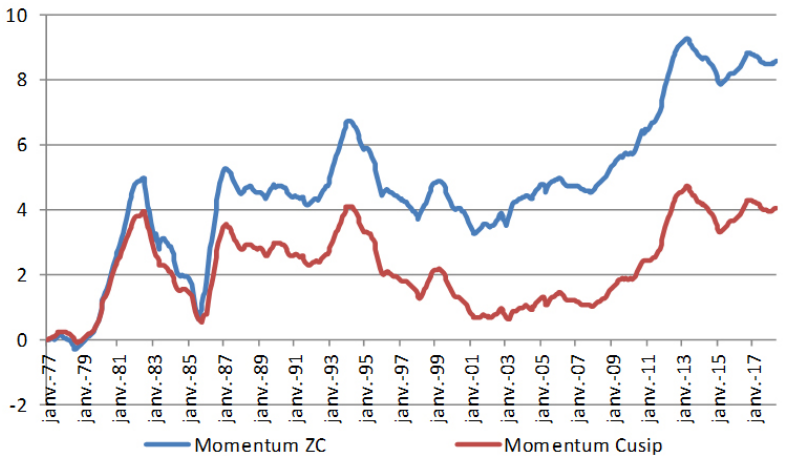
In the Introduction, we stressed the importance of using the prices of real individual Treasury bonds rather than those of synthetic zero-coupon bonds for momentum studies. As mentioned, one possible danger with discount bonds obtained from fitting models such as the Nelson-Siegel is the well-known serial autocorrelation of pricing errors, that can give rise to spurious momentum and reversal results.

We substantiate this point by following the following methodology: first we selected 9 maturities (from 2 to 10 years included) for the discount bonds. Then, on each investment date, we chose the 9 Treasury bonds with duration closest to the duration of the virtual discount bonds. Finally, we calculated the returns form the cross-sectional momentum strategies for the real and virtual portfolios, for 12-month look-back and investment horizons. (Qualitatively very similar results, not reported for the sake of brevity, were obtained for different look-back/ investment horizons.)

As Fig (3) clearly shows, the cumulative returns from the two strategies are qualitatively similar (with a correlation of 91%). However, the protability of the virtual strategy is strongly overstated, and makes the returns from cross-sectional momentum appear statistically and economically significantly different from zero, while the returns from the security-level strategy are neither.

We looked at different implementation configurations for this comparison test (eg, different criteria to choose the universe of real Treasury bonds, different look-back periods, and different strategies —eg, time series rather than cross-sectional momentum), with no qualitative difference in results. We stress that the test reported is the one for which the correspondence between the virtual and real strategies is highest. For the other congurations tested the differences were higher, and always attering the zero-discount bond strategy with respect to the real Treasury strategy.

*Figure 3: The cumulative returns from a cross-sectional momentum strategy implemented with "virtual" discount bonds (curve labelled "Momentum ZC") and with individual real securities (curve labelled "Momentum CUSIP").*



# 7. Conclusions

# 7. Conclusions

With this study, we have carried out what we believe is the most granular and extensive study of momentum in US Treasuries. We confirm previous findings that time-series momentum strategies are indeed profitable with these securities (at least for look-back and investment periods of 9 and 12 months). We also present richer results, by combining two types of time-series momentum and a cross-sectional momentum strategy, and showing that their returns must be linked by an identity. This allows us to not only to observe, but also to understand why, cross-sectional strategies with variable notionals are not profitable in US Treasuries. We show that time-series momentum strategies are robust, both in degree of protability and in lags, to different specications of the notional (eg, variable or standardised). More surprisingly, we also show that adjusting the notional of the cross-sectional strategy by duration becomes very significantly profitable over an extended range of lags (6 to 12). We explain, in Section 4, how this can be the case, and we link the protability to two concomitant factors: i) the ability of the duration-adjustment procedure to single out winners and losers by their exposure to slope changes, and ii) the degree of mean-reversion of the slope.

Finally, we have presented some long-only strategies that exploit our findings, and that significantly outperform an equal-weighted market return.

# Appendices

# Appendices

## Appendix I — 9.1 Cross-Sectional Momentum

### 9.1.1 Long-Short Framework

No Duration Adjustment We apply the empirical methodology suggested by Lewellen (2002) to build a zero-cost cross-sectional momentum strategy as follows:

1. We fix a look-back period of *L* months and a holding (investment) period of *H* months. In order to limit the possibility of data mining, we use identical look-back and holding (investment) periods. We consider four possible values for the couple (*L*, *H*): (3,3), (6,6), (9,9) and (12,12).
2. At end of month date *t*, we consider all the $N_t$ bonds that (i) are in the universe at date *t*, (ii) were in the universe at date *t*−*L* and (iii) that will be in the universe at date *t* + *H*.
3. At date *t*, we compute for each bond *i* its relative *L*-month past excess return with respect to the market : $(r^L_{i,t} - r^L_{m,t})$. $r^L_{i,t}$ is the bond *i* *L*-month past performance, and $r^L_{m,t}$ is the market L-month past performance.[5]
4. At date *t*, we assign to each bond *i* the weight $w_{i,t} = \frac{1}{N_t}\left(r^L_{i,t} - r^L_{m,t}\right)$. We have $\sum_{i=1}^{N_t} w_{i,t} = 0$.
5. Finally, we normalise the weights so as to have a cross-sectional zero-cost momentum portfolio, that is, to be 1$ long and 1$ short at the beginning of the investment period:

$$w^{norm}_{i,t} = \frac{w_{i,t}}{\sum_{i=1}^{N_t} w^+_{i,t}} \text{ where } w^+_{i,t} = w_{i,t} \text{ if } w_{i,t} > 0 \text{ and } w^+_{i,t} = 0 \text{ otherwise.}$$

With Duration Adjustment We also implement a cross-sectional duration-adjusted momentum strategy by following the same protocol as above but by duration-adjusting the notionals of the short and long positions. Duration-adjustment is achieved by dividing the returns of each bond by their duration. The duration-adjusted market return is the cross-sectional average of the duration-adjusted bond returns. Duration adjustment (*which does not imply duration neutralisation*) achieves an approximate risk parity (volatility parity) among the various constituent bonds. We normalise the weights to have a cross-sectional zero-cost momentum portfolio that is 1$ long and 1$ short at the beginning of the investment period.

### 9.1.2 Long-Only Framework

In the long-only framework, we implement a long-only version of the cross-sectional strategy, whereby we compare the returns from giving each bond in the universe an equal duration-adjusted notional with the returns from the strategies of giving equal duration-adjusted weights to the previous period winners and losers. More precisely, we build a yearly-rebalanced long-only winner portfolio as follows:

1. At inception date, we compute for each bond its duration adjusted 1-year past total return with respect to the duration-adjusted 1-year past total return of the market.
2. We only keep in the winner portfolio bonds for which the previous quantity is positive and we

5 - The market is proxied by an equal-weight portfolio of the $N_t$ bonds.

define intermediary bond weights in the winner portfolio as: $w_{i,t} = \frac{1}{N_t^+}(r_{i,t} - r_{m,t})$.
3. We finally normalise the weights such as their sum is equal to one.
4. We keep the portfolio buy-and-hold until the next rebalancing date.
5. At each rebalancing date we rebalance the portfolio by following steps 1., 2. and 3.

The procedure to build the yearly-rebalanced long-only loser portfolio is analogous.

## 9.2 Time-Series Momentum

### 9.2.1 Long-Short Zero-Cost Time-Series (Self) Momentum Strategies

We apply the following empirical protocol:
1. We fix a look-back period of *L* months and a holding period of *H* months.
2. At end of month date *t*, we consider all the $N_t$ bonds that (1) are in the universe at date *t*, (2) were in the universe at date *t*–*L* and (3) that will be in the universe at date *t* + *H*.
3. At date *t*, we compute for each bond *i* its relative *L*-month past excess return with respect to the risk-free asset.
4. At date *t*, we assign to each bond *i* the weight $w_{i,t} = \frac{1}{N_t}\left(\text{sign}\left(r_{i,t}^L\right)\right)$ and fund the position with a corresponding risk-free asset weight: $w_{i,t}^{funding} = -w_{i,t}$.

We have then: $\sum_{i=1}^{N_t}\left(w_{i,t} + w_{i,t}^{funding}\right) = 0$.

5. Finally, we invest in a time-series (self) zero-cost momentum portfolio that is 1$ long and 1$ short at the beginning of the investment period.

### 9.2.2 Long-Short Zero-Cost Time-Series (Market) Momentum Strategies

We apply the following empirical protocol:
1. We fix a look-back period of *L* months and a holding period of *H* months.
2. At end of month date *t*, we consider all the $N_t$ bonds that (1) are in the universe at date *t* and (2) were in the universe at date *t* – *L* and (3) that will be in the universe at date *t* + *H*.
3. At date *t*, we compute the market relative *L*-month past excess return with respect to the risk-free asset.
4. At date *t*, we assign to each bond *i* the weight $w_{i,t} = \frac{1}{N_t}\left(\text{sign}\left(r_{m,t}^L\right)\right)$and fund the position with a corresponding risk-free asset weight: $w_{i,t}^{funding} = -w_{i,t}$.

We have then: $\sum_{i=1}^{N_t}\left(w_{i,t} + w_{i,t}^{funding}\right) = 0$.

5. Finally, we invest in a time-series (market) zero-cost momentum portfolio that is 1$ long and 1$ short at the beginning of the investment period.

# Appendices

## Appendix II – 10.1 Preliminary Results for Momentum/Reversals Strategies

First we report a few results that will be used later.

**Result 1:** If

$$\frac{1}{N}\sum_{i=1}^{N}\mu_i = \mu_m$$

then

$$\frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 + \mu_m^2 - 2\mu_i\mu_m\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 + \mu_m^2\right) - \frac{2}{N}\left(\sum_{i=1}^{N}\mu_i\right)_{N\mu_m}\mu_m$$

$$= \left[\frac{1}{N}\sum_{i=1}^{N}\mu_i^2\right] + \mu_m^2 - 2\mu_m^2$$

$$= \left[\frac{1}{N}\sum_{i=1}^{N}\mu_i^2\right] - \mu_m^2$$

$$\left[\frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 - \mu_m^2\right)\right] \quad (3)$$

Noting that Equation (3) is equal to the cross-sectional variance of returns, we conclude that the cross-sectional variance of returns is also equal to $\frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 - \mu_m^2\right)$.

Important: the cross-sectional variance – which must be positive – is equal to $\frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 - \mu_m^2\right) > 0$, not to $\frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 - \mu_m^2\right) < 0$.

**Result 2:** From the result *cov* [*a* + *b*, *c*] = *cov* [*a*, *c*]+*cov* [*b*, *c*], the covariance between the return, $R_i$, of asset *i*, and the market return, $R_m = \frac{1}{N}\sum R_i$ is given by

$$cov\left[R_i, R_m\right]$$

$$= cov\left[R_i, \frac{1}{N}\sum_k R_k\right] = \frac{1}{N}cov\left[R_i, \sum_k R_k\right]$$

$$= \frac{1}{N}\sum_k cov\left[R_i, R_k\right] \quad (4)$$

# Appendices

## 10.2 Decomposition of TS and XS Reversals/Momentum Returns

In all the derivations we assume time homogeneity. So, for instance, for any $x_t$, $E[x_{t-k}] = E[x_t]$.

### 10.2.1 Cross-Sectional Reversals

In order to capture profits from cross-section reversal strategies, let the weights assigned to the *i*th of *N* assets be given by

$$\omega_{i,t}^{xs,r}(k) = -\frac{1}{N}(R_{i,t-k} - R_{m,t-k}) \tag{5}$$

with

$$R_{m,t-k} = \frac{1}{N}\sum R_{i,t-k} \tag{6}$$

At time *t* construct a portfolio, $\pi_t(k)$, (indexed by the time lag used to ascertain the past returns). Its return will be given by:

$$\begin{aligned}
\pi_t^{xs,r}(k) &= \sum \omega_{i,t}^{xs,r}(k) R_{i,t} \\
&= -\frac{1}{N}\sum (R_{i,t-k} - R_{m,t-k}) R_{i,t} \\
&= -\frac{1}{N}\sum R_{i,t-k}R_{i,t} + \frac{1}{N}\sum R_{m,t-k}R_{i,t} \\
&= -\frac{1}{N}\sum R_{i,t-k}R_{i,t} + R_{m,t-k}\underbrace{\frac{1}{N}\sum R_{i,t}}_{R_{m,t}} \\
&= -\frac{1}{N}\sum R_{i,t-k}R_{i,t} + R_{m,t-k}R_{m,t}
\end{aligned} \tag{7}$$

The expected portfolio return will be given by

$$E[\pi_t^{xs,r}(k)] =$$

$$\underbrace{-\frac{1}{N}\sum E[R_{i,t-k}R_{i,t}]}_{a} + \underbrace{E[R_{m,t-k}R_{m,t}]}_{b} \tag{8}$$

We start from *a*:

$$\begin{aligned}
a &= -\frac{1}{N}\sum E[R_{i,t-k}R_{i,t}] \\
&= -\frac{1}{N}\sum E[R_{i,t-k}]E[R_{i,t}] + cov[R_{i,t-k}, R_{i,t}] \\
&= -\frac{1}{N}\left(\sum \mu_i^2 + cov[R_{i,t-k}, R_{i,t}]\right)
\end{aligned} \tag{9}$$

# Appendices

Next for *b*:

$$\begin{aligned} b &= E\left[R_{m,t-k}R_{m,t}\right] \\ &= E\left[R_{m,t-k}\right]E\left[R_{m,t}\right] + cov\left[R_{m,t-k}, R_{m,t}\right] \\ &= \mu_m^2 + cov\left[R_{m,t-k}, R_{m,t}\right] \end{aligned} \tag{10}$$

Therefore for the expectation of the reversal portfolio we have

$$\begin{aligned} E\left[\pi_t^{xs,r}(k)\right] = \\ = -\frac{1}{N}\left(\sum \mu_i^2 + cov\left[R_{i,t-k}, R_{i,t}\right]\right) + \mu_m^2 + cov\left[R_{m,t-k}, R_{m,t}\right] \\ = -\frac{1}{N}\left(\sum cov\left[R_{i,t-k}, R_{i,t}\right]\right) + cov\left[R_{m,t-k}, R_{m,t}\right] - \frac{1}{N}\sum\left(\mu_i - \mu_m\right)^2 \end{aligned} \tag{11}$$

where in the last line use has been made of the result

$$\frac{1}{N}\sum_{i=1}^{N}\left(\mu_i - \mu_m\right)^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\mu_i^2 - \mu_m^2\right) \tag{12}$$

This result can be expanded as follows. Substituting $R_{m,t} = \frac{1}{N}\sum R_{i,t}$, we have

$$\begin{aligned} E\left[\pi_t^{xs,r}(k)\right] = \\ -\frac{1}{N}\left(\sum cov\left[R_{i,t-k}, R_{i,t}\right]\right) \\ +\frac{1}{N}\sum_i cov\left[R_{m,t-k}, R_{i,t}\right] \\ -\frac{1}{N}\sum\left(\mu_i - \mu_m\right)^2 \end{aligned} \tag{13}$$

and substituting again

$$R_{m,t-k} = \frac{1}{N}\sum R_{i,t-k},$$

$$E\left[\pi_t^{xs,r}(k)\right] =$$

$$-\frac{1}{N}\sum\left(\underbrace{cov\left[R_{i,t-k}, R_{i,t}\right]}_{\text{serial autocovariance}}\right)$$

$$+\frac{1}{N^2}\sum_i\sum_j \underbrace{cov\left[R_{j,t-k},R_{i,t}\right]}_{\text{serial cross-covariance}}$$
$$-\frac{1}{N}\sum \underbrace{\left(\mu_i-\mu_m\right)^2}_{\text{dispersion of returns}} \tag{14}$$

### 10.2.2 Cross-Sectional Momentum

Now we look at cross-sectional momentum. We have

$$\omega_{i,t}^{xs,m}(k)=\frac{1}{N}\left(R_{i,t-k}-R_{m,t-k}\right) \tag{15}$$

Following the same steps as above we have

$$E\left[\pi_t^{xs,m}(k)\right]=$$
$$\frac{1}{N}\sum\left(\underbrace{cov\left[R_{i,t-k},R_{i,t}\right]}_{\text{serial autocovariance}}\right)$$
$$-\frac{1}{N^2}\sum_i\sum_j \underbrace{cov\left[R_{j,t-k},R_{i,t}\right]}_{\text{serial cross-covariance}}$$
$$+\frac{1}{N}\sum \underbrace{\left(\mu_i-\mu_m\right)^2}_{\text{dispersion of returns}} \tag{16}$$

### 10.2.3 Time Series Momentum and Reversals – I

We first look at the situation where, for each asset, the strategy takes a long (momentum) or short (reversals) position if the same asset had a positive return in the previous *k*-month period (and conversely for a negative return). *The size of the position in a given security depends on how large the positive (or negative) return for the same security was in the pervious period.*

In this case, we have

$$\omega_{i,t}^{ts,m_1}(k)=\frac{1}{N}R_{i,t-k} \tag{17}$$

and therefore the profit from investing with different weights in all the securities is given by

$$\pi_t^{ts,m1}(k)=\frac{1}{N}\sum R_{i,t-k}R_{i,t} \tag{18}$$

The expectation is

$$E\left[\pi_t^{ts,m1}(k)\right] = \frac{1}{N}\sum E\left[R_{i,t-k}R_{i,t}\right]$$

$$= \frac{1}{N}\left(\sum \underbrace{E\left[R_{i,t-k}\right]}_{\mu_i}\underbrace{E\left[R_{i,t}\right]}_{\mu_i} + cov\left[R_{i,t-k}, R_{i,t}\right]\right)$$

$$= \frac{1}{N}\sum\left(\mu_i^2 + cov\left[R_{i,t-k}, R_{i,t}\right]\right) \tag{19}$$

For reversals with the same weight allocation rule we have

$$\omega_{i,t}^{ts,r_1}(k) = -\frac{1}{N}R_{i,t-k} \tag{20}$$

and therefore

$$\pi_t^{ts,r1}(k) = -\frac{1}{N}\sum R_{i,t-k}R_{i,t} \tag{21}$$

and

$$E\left[\pi_t^{ts,r1}(k)\right] =$$

$$-\frac{1}{N}\sum\left(\mu_i^2 + cov\left[R_{i,t-k}, R_{i,t}\right]\right) \tag{22}$$

### 10.2.4 Time Series Momentum and Reversals – II

Consider now a different momentum or reversal strategy. Starting from momentum, we go long (short) every asset with the same weight if the market portfolio return was positive (negative). T*he size of the position in a given security depends on how large the positive (or negative) return for the whole market was in the pervious period.* So, we have for momentum:

$$\omega_{i,t}^{ts,m2}(k) = \frac{1}{N}R_{m,t-k}$$

and

$$\pi_t^{ts,m2}(k) = \frac{1}{N}R_{m,t-k}\sum R_{i,t} \tag{23}$$

Therefore

$$E\left[\pi_t^{ts,m2}(k)\right] = \frac{1}{N}E\left[R_{m,t-k}\sum R_{i,t}\right]$$

$$= \frac{1}{N}\left(E\left[R_{m,t-k}\right]E\left[\sum R_{i,t}\right] + cov\left[\sum R_{m,t-k}, R_{i,t}\right]\right)$$

$$= \mu_m\frac{1}{N}\sum\mu_i + \frac{1}{N}cov\left[\sum R_{m,t-k}, R_{i,t}\right]$$

$$= \mu_m^2 + \frac{1}{N}cov\left[\sum R_{m,t-k}, R_{i,t}\right]$$

$$= \mu_m^2 + \frac{1}{N^2}\sum_{ij} cov\left[R_{j,t-k}, R_{i,t}\right] \tag{24}$$

For market-based time series reversals we have

$$\omega_{i,t}^{ts,r2}(k) = -\frac{1}{N}R_{m,t-k} \tag{25}$$

and

$$\pi_t^{ts,r2}(k) = -\frac{1}{N}R_{m,t-k}\sum R_{i,t} \tag{26}$$

$$E\left[\pi_t^{ts,r2}(k)\right] = -\mu_m^2 - \frac{1}{N^2}\sum_{ij} cov\left[R_{j,t-k}, R_{i,t}\right] \tag{27}$$

## 10.3 Comparing Different Strategies

Putting all the results together we have for reversals

$$E\left[\pi_t^{xs,r}(k)\right] =$$

$$\underbrace{-\frac{1}{N}\sum cov\left[R_{i,t-k}, R_{i,t}\right]}_{\alpha} + \underbrace{\frac{1}{N^2}\sum_{i,j} cov\left[R_{j,t-k}, R_{i,t}\right]}_{\beta} - \underbrace{\frac{1}{N}\sum\left(\mu_i^2\right)}_{\gamma} + \mu_m^2$$

$$E\left[\pi_t^{ts,r1}(k)\right] = \underbrace{-\frac{1}{N}\sum\mu_i^2}_{\gamma} \underbrace{-\frac{1}{N}\sum cov\left[R_{i,t-k}, R_{i,t}\right]}_{\alpha}$$

$$E\left[\pi_t^{ts,r2}(k)\right] = -\mu_m^2 \underbrace{-\frac{1}{N^2}\sum_{ij} cov\left[R_{j,t-k}, R_{i,t}\right]}_{\beta}$$

with the similar expressions for momentum. Therefore we have

$$E\left[\pi_t^{xs,r}(k)\right] = -\alpha + \beta - \gamma + \mu_m^2 \tag{28}$$

$$E\left[\pi_t^{ts,r1}(k)\right] = -\alpha - \gamma \tag{29}$$

$$E\left[\pi_t^{ts,r2}(k)\right] = -\beta - \mu_m^2 \tag{30}$$

and

$$\begin{aligned} & E\left[\pi_t^{ts,r1}(k)\right] - E\left[\pi_t^{ts,r2}(k)\right] \\ &= \underbrace{-\alpha - \gamma}_{E\left[\pi_t^{ts,r1}(k)\right]} + \underbrace{\beta + \mu_m^2}_{-E\left[\pi_t^{ts,r2}(k)\right]} \\ &= E\left[\pi_t^{xs,r}(k)\right] \end{aligned} \tag{31}$$

Equivalently, we have

$$E\left[\pi_t^{xs,r}(k)\right] - E\left[\pi_t^{ts,r1}(k)\right] + E\left[\pi_t^{ts,r2}(k)\right] = 0\ . \tag{32}$$

This can be re-written as

$$E\left[\pi_t^{xs,r}(k)\right] = E\left[\pi_t^{ts,r1}(k)\right] - E\left[\pi_t^{ts,r2}(k)\right] \tag{33}$$

which establishes the relationship between time-series and cross-sectional strategies that we set out to establish, and which shows that, for each lag, the profitability of the cross-sectional strategy is given by the difference in protability of the two time series strategies.

## 10.4 Differences in expected returns

What is this difference between the expectation of the two time-series reversal strategies? This is given by

$$\begin{aligned} & E\left[\pi_t^{ts,r2}(k)\right] - E\left[\pi_t^{ts,r1}(k)\right] \\ &= \underbrace{\left[\frac{1}{N}\sum \mu_i^2 - \mu_m^2\right]}_{x\sec var} + \left(\frac{1}{N}\sum_i cov\left[R_{i,t-k}, R_{i,t}\right] - \frac{1}{N^2}\sum_{ij} cov\left[R_{j,t-k}, R_{i,t}\right]\right) \end{aligned}$$

$$= x \sec var + \left( \frac{1}{N} \sum_i cov\left[R_{i,t-k}, R_{i,t}\right] - \frac{1}{N^2} \sum_{i,j} cov\left[R_{j,t-k}, R_{i,t}\right] \right)$$

$$= x \sec var + \left( \frac{1}{N} \sum_i cov\left[R_{i,t-k}, R_{i,t}\right] - \frac{1}{N} \sum_i \left( \frac{1}{N} \sum_j cov\left[R_{j,t-k}, R_{i,t}\right] \right) \right) \quad (34)$$

Let's look at the term $\frac{1}{N} \sum_i \left( \frac{1}{N} \sum_j cov\left[R_{j,t-k}, R_{i,t}\right] \right)$ in detail. This is given by:

- the average of the cross-covariance between stock *i* and all the other stocks
- the average of this quantity across all the *i* stocks.

Therefore, the difference between the two time series strategies is given by the sum of the cross-sectional variance of returns plus the difference between

- the average serial auto-covariance of each individual security (averaged across all stocks), minus
- the doubly averaged cross-covariance between every pair of securities (averaged across all pairs).

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# About Amundi ETF, Indexing and Smart Beta

# About Amundi ETF, Indexing and Smart Beta

With more than €112 billion[1] in assets under management, Amundi ETF, Indexing and Smart Beta is one of Amundi's strategic business areas and is a key growth driver for the Group.

Amundi ETF, Indexing and Smart Beta business line provides investors - whether institutionals or distributors - with robust, innovative, and cost-efficient solutions, leveraging Amundi Group's scale and large resources. The platform also offers investors fully customized solutions (ESG, Low Carbon, specific exclusions, risk constraints, etc.).

With over 30 years of benchmark construction and replication expertise, Amundi is a trusted name in ETF & Index management among the world's largest institutions. The team is also recognized for its ability to develop Smart Beta & Factor Investing solutions, with more than 10-year track-record.

1- All figures and data are provided by Amundi ETF, Indexing & Smart Beta at end March 2019

# About EDHEC-Risk Institute

# About EDHEC-Risk Institute

**Founded in 1906, EDHEC is one of the foremost international business schools. Operating from campuses in Lille, Nice, Paris, London and Singapore, EDHEC is one of the top 15 European business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This Institute boasts a team of permanent professors, engineers and support staff, and counts a large number of affiliate professors and research associates from the financial industry among its ranks.**

## The Need for Investment Solutions and Risk Management

Investment management is justified as an industry only to the extent that it can demonstrate a capacity to add value through the design of dedicated and meaningful investor-centric investment solutions, as opposed to one-size-fits-all manager-centric investment products. After several decades of relative inertia, the much needed move towards investment solutions has been greatly facilitated by a true industrial revolution triggered by profound paradigm changes in terms of (1) mass production of cost- and risk-efficient smart factor indices; (2) mass customisation of liability-driven investing and goal-based investing strategies; and (3) mass distribution, with robo-advisor technologies. In parallel, the investment industry is strongly impacted by two other major external revolutions, namely the digital revolution and the environmental revolution.

In this fast-moving environment, EDHEC-Risk Institute positions itself as the leading academic think-tank in the area of investment solutions, which gives true significance to the investment management practice. Through our multi-faceted programme of research, outreach, education and industry partnership initiatives, our ambition is to support industry players, both asset owners and asset managers, in their efforts to transition towards a novel, welfare-improving, investment management paradigm.

## EDHEC-Risk New Initiatives

In addition to the EDHEC Alternative Indexes, which are used as performance benchmarks for risk analysis by investors in hedge funds, and the EDHEC-IEIF Monthly Commercial Property index, which tracks the performance of the French commercial property market through SCPIs, EDHEC-Risk has recently launched a series of new initiatives.

- The EDHEC-Princeton Retirement Goal-Based Investing Index Series, launched in May 2018, which represent asset allocation benchmarks for innovative mass-customised target-date solutions for individuals preparing for retirement;

- The EDHEC Bond Risk Premium Monitor, the purpose of which is to offer to investment and academic communities a tool to quantify and analyse the risk premium associated with Government bonds;

- The EDHEC-Risk Investment Solutions (Serious) Game, which is meant to facilitate engagement with graduate students or investment professionals enrolled on one of EDHEC-Risk's various campus-based, blended or fully-digital educational programmes.

# About EDHEC-Risk Institute

## Academic Excellence and Industry Relevance

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Seven research programmes have been conducted by the centre to date:

- Investment Solutions in Institutional and Individual Money Management;
- Equity Risk Premia in Investment Solutions;
- Fixed-Income Risk Premia in Investment Solutions;
- Alternative Risk Premia in Investment Solutions;
- Multi-Asset Multi-Factor Investment Solutions;
- Reporting and Regulation for Investment Solutions;
- Technology, Big Data and Artificial Intelligence for Investment Solutions.

EDHEC-Risk Institute's seven research programmes explore interrelated aspects of investment solutions to advance the frontiers of knowledge and foster industry innovation. They receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in the City of London (United Kingdom) and Nice, (France).

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- **Financial Risk Management as a Source of Performance,**
in partnership with the *French Asset Management Association (Association Française de la Gestion financière – AFG)*;
- **ETF, Indexing and Smart Beta Investment Strategies,**
in partnership with *Amundi*;
- **Regulation and Institutional Investment,**
in partnership with *AXA Investment Managers*;
- **Optimising Bond Portfolios,**
in partnership with *BDF Gestion*;
- **Asset-Liability Management and Institutional Investment Management,**
in partnership with *BNP Paribas Investment Partners*;
- **New Frontiers in Risk Assessment and Performance Reporting,**
in partnership with *CACEIS*;
- **Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation,**
in partnership with *CME Group*;

# About EDHEC-Risk Institute

- **Asset-Liability Management Techniques for Sovereign Wealth Fund Management,**
in partnership with *Deutsche Bank*;
- **The Benefits of Volatility Derivatives in Equity Portfolio Management,**
in partnership with *Eurex*;
- **Innovations and Regulations in Investment Banking,**
in partnership with the *French Banking Federation (FBF)*;
- **Dynamic Allocation Models and New Forms of Target-Date Funds for Private and Institutional Clients,**
in partnership with *La Française AM*;
- **Risk Allocation Solutions,**
in partnership with *Lyxor Asset Management*;
- **Infrastructure Equity Investment Management and Benchmarking,**
in partnership with *Meridiam and Campbell Lutyens*;
- **Risk Allocation Framework for Goal-Driven Investing Strategies,**
in partnership with *Merrill Lynch Wealth Management*;
- **Financial Engineering and Global Alternative Portfolios for Institutional Investors,**
in partnership with *Morgan Stanley Investment Management*;
- **Investment and Governance Characteristics of Infrastructure Debt Investments,**
in partnership with *Natixis*;
- **Advanced Investment Solutions for Liability Hedging for Inflation Risk,**
in partnership with *Ontario Teachers'* Pension Plan;
- **Cross-Sectional and Time-Series Estimates of Risk Premia in Bond Markets,**
in partnership with *PIMCO*;
- **Active Allocation to Smart Factor Indices,**
in partnership with *Rothschild & Cie*;
- **Solvency II,**
in partnership with *Russell Investments*;
- **Advanced Modelling for Alternative Investments,**
in partnership with *Société Générale Prime Services (Newedge)*;
- **Structured Equity Investment Strategies for Long-Term Asian Investors,**
in partnership with *Société Générale Corporate & Investment Banking*.

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, https://risk.edhec.edu, which is devoted to international risk and investment management research for the industry. The website is aimed at professionals who wish to benefit from EDHEC-Risk's analysis and expertise in the area of investment solutions. Its quarterly newsletter is distributed to more than 150,000 readers.

# About EDHEC-Risk Institute

## Research for Business

EDHEC-Risk Institute also has highly significant executive education activities for professionals, in partnership with prestigious academic partners. EDHEC-Risk's executive education programmes help investment professionals upgrade their skills with advanced asset allocation and risk management training across traditional and alternative classes.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements. The first was with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of investment solutions for institutions and individuals. The second was with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up ERI Scientific Beta, which is an original initiative that aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

EDHEC-Risk Institute also contributed to the 2016 launch of EDHEC Infrastructure Institute (EDHEC*infra*), a spin-off dedicated to benchmarking private infrastructure investments. EDHEC*infra* was created to address the profound knowledge gap faced by infrastructure investors by collecting and standardising private investment and cash flow data and running state-of-the-art asset pricing and risk models to create the performance benchmarks that are needed for asset allocation, prudential regulation and the design of infrastructure investment solutions.

# EDHEC-Risk Institute Publications and Position Papers (2016-2019)

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## 2019

- Maeso, J.M., Martellini, L. and R. Rebonato. Cross-Sectional and Time-Series Momentum in Sovereign Bond Markets (May).
- Maeso, J.M., Martellini, L. and R. Rebonato. Defining and Expoiting Value in Sovereign Bond Market (May).
- Maeso, J.M., Martellini, L. and R. Rebonato. Factor Investing in Sovereign Bond Markets - Time-Series Perspective (May).

## 2018

- Goltz, F. and V. Le Sourd. The EDHEC European ETF and Smart Beta and Factor Investing Survey 2018 (August).
- Mantilla-Garcia, D. Maximising the Volatility Return: A Risk-Based Strategy for Homogeneous Groups of Assets (June).
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## 2017

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- Martellini, L. and V. Milhau. Mass Customisation versus Mass Production in Retirement Investment Management. Addressing a "Tough Engineering Problem" (May).
- Esakia, M., F. Goltz, S. Sivasubramanian and J. Ulahel. Smart Beta Replication Costs (February).
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## 2016

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