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Exchange-Traded Fixed-Income Derivatives in Asset Management and Asset-Liability Management

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Abstract

In this paper, we examine how standard exchange-traded fixed-income derivatives (futures and options on futures contracts) can be made part of sound risk and asset management in such a way as to improve the risk and return performance characteristics of managed portfolios. Our results show that the non-linear character of the returns on protective option strategies offers appealing risk reduction properties in pure asset management. Consequently, such strategies should optimally receive a significant allocation, especially when investors are concerned with minimising extreme risks. In asset-liability management, we also show that fixed-income derivatives in general, and recently launched long-term futures contracts in particular, offer significant shortfall risk reduction benefits. These results have potentially significant implications in the context of an increased focus on matching liability portfolios.

Introduction

Interest in the use of fixed-income derivatives in asset management has perhaps never been so great, both from a pure asset management perspective and from an asset-liability management perspective. For one, from a pure asset management perspective, historically low levels of long-term yields, combined with the prospect of increases in short-term rates by central banks in Europe as well as in the United States, are incentives to implement strategies that aim to deliver downside protection by combining a long position in bond markets with a long position in put options. For another, recent pension fund shortfalls have drawn attention to the risk management practices of institutional investors and defined benefit pension plans, and have given institutional investors new incentives to use fixed-income derivatives in the management of their liability risk.

A so-called perfect storm of adverse market conditions around the turn of the millennium has devastated many corporate defined benefit pension plans. Negative equity market returns have eroded plan assets at the same time as declining interest rates have increased the mark-to-market value of benefit obligations and contributions. In extreme cases, corporate pension plans have been left with funding gaps as large as, or larger than, the market capitalisation of the plan sponsor. For example, in 2003, the companies included in the S&P 500 and the FTSE 100 index faced a cumulative deficit of $225 billion and £55 billion respectively (as per 2003 surveys by Credit Suisse First Boston and Standard Life Investments), while the worldwide deficit reached an estimated $1,500 to $2,000 billion (Watson Wyatt 2003). That institutional investors and especially pension funds have been so dramatically affected by recent market downturns has highlighted the weakness of risk management practices. In particular, it has been argued that asset allocation strategies commonly used, which were heavily skewed towards equities, were not, in the absence of any protection with respect to their downside risk, consistent with sound liability risk management. On the heels of recent changes in accounting standards and regulations that have led to an increased focus on liability risk management, new approaches, referred to as liability-driven investment, have rapidly attracted the interest of pension funds, insurance companies, and investment consultants. Such strategies, which can add significant value in terms of liability risk management, lead to an increased focus on matching investors' portfolios via cash instruments (long-term real and nominal bonds) or via derivative instruments.

Although an impressive amount of research has been done on issues related to bond and fixed-income derivative pricing and hedging, relatively few results can actually be found on how these instruments can be used to improve an investor's welfare in the context of portfolio strategies. The focus of this paper then is on how various types of derivative securities can be used to shift the risk associated with investing in fixed-income securities. We take an asset allocation perspective and assess in greater detail the benefits of including bond futures and options on these futures in a fixed-income portfolio. In addition to linear instruments such as futures, options have non-linear payoffs that allow investors to construct funds with skewed or dissymmetric return profiles. In comparison to simple bond index exposure, these may offer an efficient way to limit the extreme values of a portfolio return distribution.

As such, our paper is closely related to a number of papers that look at optimal portfolios when investors have access to an asset and options on this asset. It is possible to take Merton's (1973) replicating portfolio interpretation of the Black and Scholes (1973) option pricing model and argue that options are redundant assets and that, as a result, the optimal allocation to such assets should be either indeterminate (when an investor's expectation of parameter values match those of the markets) or infinite (when the investor's beliefs lead him to view the derivative asset as underpriced). But it must be acknowledged that this result depends on the concept of dynamic completeness (see Radner 1972 for a formal analysis), which can be justified only under a set of rather stringent assumptions that are never met in practice (continuous trading, the absence of transaction costs, leverage or short-sales constraints, constant volatility and drift of the underlying asset, symmetric information, and so on). When any of these assumptions are relaxed, it can actually be shown that the introduction of derivatives would improve the welfare of investors. A first strand of the literature, initiated by Brennan and Solanki (1981), has examined the question of optimal positioning in derivative assets from the static investor's standpoint. The question there is to determine the general shape of the non-linear function of the underlying asset that would generate the highest level of expected utility for a given investor. Carr and Madan (2001) extend this early work by considering optimal portfolios of a riskless asset, a risky asset, and derivatives of the latter in an expected utility framework. A second strand of the literature (starting with Leland 1980 and expanded on by Benninga and Blume 1985) has studied the question from a different angle: taking as given a set of standard derivatives, these papers examine the features of (static) investors' preferences that would support a rational holding of these contracts.

This paper, which distinguishes itself by focusing exclusively on bond derivatives, complements existing literature by focusing on the use of derivatives, not only in asset management, but also from an asset-liability management perspective. In an attempt to focus on plain vanilla products that can readily be used by investors, we study a set of fixed-income derivatives traded on Eurex, the leading futures and options market for euro-denominated derivative instruments. The use of futures and options will be organised according to the investment process and objectives and we will examine the value of introducing derivatives both in a pure asset management context and in an asset-liability management context.

The rest of the paper is organised as follows: section one introduces the theoretical model and shows how we implement it. Section two details the implementation of the investment strategies and assesses their benefits in a pure asset management context. Section three examines similar questions but from an asset-liability management (ALM) perspective.

1. The Framework

To assess the different uses and benefits of derivatives in fixed-income portfolios in the context of a full-blown numerical experiment, it is necessary to generate stochastic scenarios for asset prices. The scenario generation process will be similar for the whole paper, with additional assumptions on investors' preferences needed for some sections. In this introductory section, we therefore describe our assumptions with respect to the modelling of fixed-income markets.

1.1.Choice of a Model

Modelling future uncertainty is critical to any investment management problem. A stochastic forecasting model consists of multivariate time series models. While generating stochastic scenarios for stock prices can be achieved in a straightforward manner, a specific challenge related to simulating bond prices is the need to ensure the absence of arbitrage between the prices of bonds with different maturities at a given point in time. In other words, one needs to impose some restrictions on the cross-section of bond prices. This can be achieved with a consistent model of the term structure of interest rates. Term structure models imply that changes in yields of bonds of different maturities can be expressed as a function of changes in some underlying factors.

The first option would be to use an equilibrium model of the term structure. This type of model starts with diffusion processes for some state variables and specifies endogenously a dynamic process for the short rate as well as a zero coupon curve. The obvious drawback is that the simulated bond prices do not automatically match those observed in the market. Arbitrage models, on the other hand, take as given the observed term structure of interest rates, which is then regarded as the underlying asset (see Ho and Lee 1986 for a discrete-time example, or Heath, Jarrow and Morton 1990, 1992 for a general formulation in continuous time).

Since we do not deal with the problem of minimising pricing errors in a practical hedging exercise, the drawback of equilibrium models is irrelevant. In fact, with our particular requirements, the calibration of popular arbitrage models would involve too much complexity for the effects that the model should capture.

These particular requirements are: i) stochastic volatility for bond prices, ii) non-redundancy of multiple bonds, and iii) the availability of closed-form pricing formulas. Particular emphasis is placed on the fact that the aim of our model is to capture the stochastic volatility of bond prices, which is an important ingredient in the analysis of options available on bond and bond futures contracts. Another reason to use a stochastic volatility model is that the presence of time-varying volatility of asset prices has been well documented in empirical studies (Bollerslev et al 2000, Reilly et al 2000, and Jones et al 1998). We must also avoid redundancy of multiple bonds, which rules out single-factor term-structure models, where bonds with different maturities are perfectly correlated. Finally, our task of testing different investment strategies will be greatly eased in terms of computational burden and complexity if we have closed-form solutions.

The above considerations led us to choose the multifactor term structure model by Longstaff and Schwartz (1992), henceforth LS. Their model i) has stochastic volatility of bond prices stemming from the stochastic volatility of the process for the short rate, ii) incorporates more than one factor, which means that bonds of different maturities are not necessarily (dynamically) redundant assets, and iii) is tractable in the sense that it allows one to obtain an explicit closed-form solution for bonds of different maturities and for options on these bonds.

The LS model is a two-factor extension of the Cox, Ingersoll, and Ross general equilibrium model. The dynamics of the short rate are similar to those in the CIR model, except for an additional factor of uncertainty given by the volatility of the short rate. The above equations make it possible to write the processes for the short rate r and the volatility of the short rate V as:

$$dr_t = \left(\alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r_t - \frac{\xi - \delta}{\beta - \alpha} V_t \right) dt + \alpha \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} dZ_t^{(1)} + \beta \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} dZ_t^{(2)}$$

$$dV_t = \left(\alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r_t - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V_t \right) dt + \alpha^2 \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} dZ_t^{(1)} + \beta^2 \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} dZ_t^{(2)}$$

where $Z^{(1)}$ and $Z^{(2)}$ are standard Brownian motions, and α, β, γ, δ, η, ξ are constant parameters.

LS obtain the price of a zero coupon bond with time to maturity τ as functions of *r* and *V*.

$$B_t(\tau, r_t, v_t) = A^{2\gamma}(\tau) B^{2\eta}(\tau) \exp(\kappa\tau + C(\tau) r_t + D(\tau) v_t)$$

where

$$\upsilon = \lambda + \xi, \qquad A(\tau) = \frac{2\phi}{(\delta + \phi)(\exp(\phi\tau) - 1) + 2\phi}$$

$$\phi = \sqrt{2\alpha + \delta^2}$$

$$\psi = \sqrt{2\beta + \upsilon^2} \qquad B(\tau) = \frac{2\psi}{(\upsilon + \psi)(\exp(\psi\tau) - 1) + 2\psi}$$

$$\kappa = \gamma(\delta + \phi) + \eta(\upsilon + \psi) \qquad C(\tau) = \frac{\alpha\phi(\exp(\psi\tau) - 1)B(\tau) - \beta\psi(\exp(\phi\tau) - 1)A(\tau)}{\phi\psi(\beta - \alpha)}$$

$$D(\tau) = \frac{\psi(\exp(\phi\tau) - 1)A(\tau) - \phi(\exp(\psi\tau) - 1)B(\tau)}{\phi\psi(\beta - \alpha)}$$

and λ is the market price of risk.

LS also obtain the price of a call option on the zero coupon bond with payoff at expiration of max(0, *F(r,V,T) – K*) as

$$C(r, V, \tau, K, T) = F(r, V, \tau, +T)\Psi(\theta_1, \theta_2; 4\gamma, 4\eta, \varpi_1, \varpi_2) - KF(r, V, \tau)\Psi(\theta_3, \theta_4; 4\gamma, 4\eta, \varpi_3, \varpi_4),$$

where

$$\theta_1 = \frac{4\zeta\phi^2}{\alpha(\exp(\phi\tau) - 1)^2 A(\tau + T)}$$

$$\theta_2 = \frac{4\zeta\psi^2}{\beta(\exp(\psi\tau) - 1)^2 B(\tau + T)}$$

$$\theta_3 = \frac{4\zeta\phi^2}{\alpha(\exp(\phi\tau) - 1)^2 A(\tau) A(T)}$$

$$\theta_4 = \frac{4\zeta\psi^2}{\beta(\exp(\psi\tau) - 1)^2 B(\tau) B(T)}$$

and

$$\varpi_1 = \frac{4\phi \exp(\phi\tau)A(\tau+T)(\beta r - V)}{\alpha(\beta-\alpha)(\exp(\phi\tau)-1)A(T)}$$

$$\varpi_2 = \frac{4\psi \exp(\psi\tau)B(\tau+T)(V-\alpha r)}{\beta(\beta-\alpha)(\exp(\psi\tau)-1)B(T)}$$

$$\varpi_3 = \frac{4\phi \exp(\phi\tau)A(\tau)(\beta r - V)}{\alpha(\beta-\alpha)(\exp(\phi\tau)-1)}$$

$$\varpi_4 = \frac{4\psi \exp(\psi\tau)B(\tau)(V-\alpha r)}{\beta(\beta-\alpha)(\exp(\psi\tau)-1)}$$

$$\zeta = \kappa T + 2\gamma \ln A(T) + 2\eta \ln B(T) - \ln K$$

The function $\Psi(\theta_1, \theta_2, 4\gamma, 4\eta, \varpi_1, \varpi_2)$ is the joint distribution

$$\int_0^{\theta_1} \int_0^{\theta_2-\theta_2 u/\theta_1} \chi^2(u;4\gamma,\varpi_1)\chi^2(\upsilon;4\eta,\varpi_2)d\upsilon du,$$

where $\chi^2(\,.\,;p,q)$ denotes the non-central chi-square density with degrees of freedom parameter p and non-centrality parameter q. We rewrite this as a single integral by applying the Fubini theorem and solve the resulting integral numerically.

1.2. Scenario Generation

The model we use for interest rate dynamics involves quite a few parameters. One approach would involve estimating these parameters by fitting the model to historical data. We choose instead to select parameter values consistent with those found in the existing literature. Table 1 contains information about the parameter values we use and about those used in other studies.

Table 1: Parameter Values.[1]

	Jensen (2001)	Longstaff and Schwartz (1992)	Longstaff and Schwartz (1993)	Navas (2004)	Munk (2002)	This Paper
Alpha	0.0001	-0.0439	0.001149	2.6693E-06	0.01	0.0001
Beta	0.00186	0.0814	0.1325	8.3633E-05	0.08	0.00186
Gamma	70.3324		3.0493	12.3990	0.1	70.3324
Delta	1.0872	0.3299	0.05658	9.0490E-0.4	0.33	1.0872
Eta	8.7990		0.1582	0.0418	16	8.7990
Xi	0.8081		3.998	4.8949E0.3	14	0.8081
Lambda				-0.2 to -0.4	0	-0.4
Nu = Xi+Lambda				- 14.4227		- 14

We adopt the parameters in Jensen (2001). These were estimated using the efficient method of moments from a sample of T-Bill rates that spans a recent period. The parameters are estimated from data with weekly frequency, which corresponds to the frequency we choose for our discretisation. In addition to the parameters in Jensen, we must specify the risk aversion parameter Lambda. We choose lambda equal to -0.4, which is in the range of parameters estimated by Navas (2004). In addition, our choice of lambda allows us to obtain returns for the bonds that correspond to recent data for German government bonds. For example, the iboxx Euro Sovereign Germany index for long maturity bonds shows an annualised return of 5.12% since its inception in 1999, which is in line with the mean return of 4.96% which we obtain in our scenarios.

Using these parameter values, which are depicted on the right-most column of Table 1, we simulate 1,000 paths for the short rate r and its volatility V according to the equations for dr and dV in the LS model. The next step is the time-discretisation of these processes for simulating paths so that we can generate scenarios to represent future uncertainty. We generate paths with 52 observations per year over a horizon of one year, and choose starting values for interest rate and interest rate volatility to be equal to their long-term mean given our chosen

1 - The parameter values from Jensen (2001) are from Table 5. Longstaff and Schwartz (1992) only estimate four parameters, since this is sufficient if bond maturity is assumed to be given. Parameter estimates are from their Table II. Parameter values from Longstaff and Schwartz (1993) are from Exhibit 3. Those from Navas (2004) are from table 3 and Figure 2. Parameter values used by Munk (1999) are from page 173.

parameter values. We obtain the long-term mean by determining the equilibrium point of the differential equations for r and V above. With this approach, we obtain an initial value of 2.1% for the interest rate and of 1.654% for interest rate volatility.

Once the paths for interest rates and interest rate volatility are simulated, returns for different asset classes are calculated on each path in order to generate the scenarios. We describe the assets in the following subsection.

1.3. Asset Universe

For reasons outlined in the introduction, we use as benchmark instruments Eurex futures contracts on notional debt instruments issued by the Federal Republic of Germany with different remaining terms, as well as options on these futures contracts.

To model the price evolution of these instruments, we generate scenarios for the price of a zero-coupon notional bond with a long-term maturity. The position in this fictitious bond is rolled over every three months. As a result, the maturity of the strategy does not decline constantly, but is instead reset every three months. We also consider options on this notional bond.[2]

1.3.1. Futures

In this section, we consider the futures contract on a long-term instrument with a six percent annual coupon (Bund futures). In section three, we will introduce an additional contract on very long-term bonds with a four percent annual coupon (Buxl futures).

We want to ensure that the instruments we model have a sensitivity to interest rate changes that corresponds to that of the underlyings of the Eurex futures. Since the duration of a pure discount bond is equal to its maturity, we use the calculated duration of the underlyings to specify the maturity of the bonds in our simulation model. For the sake of simplicity, we calculate duration as

$$D = \frac{\sum_{t=1}^{T} \frac{tC(t)}{(1+r)^t}}{P_\tau}$$

where P_τ is the bond price at expiration of the future, *C(t)* is the cash flow in period *t* and *r* is the discount rate. For the purpose of calculating duration, we set P_τ to the nominal value and *r* to the coupon rate.

Table 2 gives an overview of the contract specifications and the calculated duration.[3]

Table 2: Contract specifications

Contract	Remaining term defined in contract	Remaining term assumed by us	Calculated duration used in this paper
Euro-Bund Futures (FGBL)	8.5 to 10.5	10	7.8017
Euro-Buxl Futures (FGBX)	24 to 35	30	17.984

1.3.2. Options on Futures

We consider the Eurex options on the Bund futures contract. These options are available with expiry dates up to 6 months in the future. Available expiry dates are the three nearest calendar months, as well as the following quarterly month of the March, June, September and December cycle thereafter. A number of strike prices are available with intervals of 0.5 points or 50 ticks. For each contract month, there are at least nine call and nine put series, each with an at-the-money strike price as well as four in-the-money and four out-of-the money strike prices. There are currently no options available on the Buxl futures contract.

2 - In practice, using futures contracts and options on futures is usually considered as a cheaper alternative to the use of bond and bond options. The reason why we have chosen to model bond and bond option prices, as opposed to futures and futures option prices, is because of the increased tractability required for the numerical exercises performed below. In particular, in the Longstaff and Schwartz framework outlined above, we are able to use closed-form solutions for zero coupon bonds and options on these bonds. It should be stressed that this is done for modelling reasons only and has no conceptual consequences. Because of only minor differences in the contractual payoff of these contracts, our conclusions can be considered as directly carrying over to the use of bond futures and options on these futures.

3 - Of course, the results of the numerical exercise could easily be adjusted to reflect the exact characteristics of the actual contracts on a given trading date.

2. Derivatives Strategies in Fixed Income Portfolio Management

The management of fixed-income portfolios is widely varied, involving both passive (indexing) and active (bond-timing and bond-picking) tasks. Derivatives can in fact facilitate all of these tasks. In particular, futures on fixed-income instruments help investors or managers neutralise biases in terms of factor exposure (changes in the level or slope of the yield curve) that may result from bond-picking bets. Conversely, an investor may decide to protect himself from market risk in order to conserve only the return linked to the active bets taken by a manager. Another natural use of bond futures is for timing strategies between different maturity segments of the bond market. In addition, it is possible to create leveraged positions with futures, using arbitrage strategies between the futures contract and the underlying or hedging interest rate risk.

In what follows, we focus on how options, thanks to their ability to generate non-linear, convex payoffs that offer downside risk protection, can improve risk management. Most fixed-income strategies rely on static exposure to the returns of the different instruments. In fact, it can be argued that the implementation of dynamic strategies runs up against a number of hurdles stemming from the particular characteristics of bond markets (infrequent pricing and low liquidity compared to stock and currency markets). In these circumstances, using derivatives is an alternative to the direct implementation of dynamic trading strategies. In fact, from the derivation of no-arbitrage pricing formulas, it is well known that a payoff that is a non-linear function of the price of the underlying asset may be achieved with appropriate dynamic trading strategies in complete markets. This logic can be inversed in order to examine the equivalence of non-linear payoffs at a point in time with a dynamic trading strategy over time (Haugh and Lo 2001). From the investor's perspective, it is interesting to assess the expected benefits may of investing in such instruments.

In a first subsection, we analyse the benefits of option-based strategies on a stand-alone basis. In the next subsection, we consider the same question from a portfolio standpoint.

2.1. Option Strategies

2.1.1. Description of Strategies

A favourite strategy with investors and asset managers alike is protective put buying (PPB). This strategy consists of a long position in the underlying asset and a long position in a put option, which is rolled over as the option expires. PPB is different from portfolio insurance, since the put is rolled over in each sub-period. Therefore, the payoff at the end of a total period with multiple sub-periods does not simply correspond to a guaranteed minimum payoff, as in the case of portfolio insurance. At the end of every sub-period, however, the long position in the put option offers protection from downside risk; the left tail of the returns distribution is avoided.

PPB has been widely studied in the context of equity portfolio management (Merton et al 1982, Figlewski et al 1993). For fixed-income derivatives, however, it has not been analysed in any detail. This neglect is surprising, as downside risk is no less significant in fixed-income markets. In the theoretical setup that we have chosen, changes in the short rate and changes in interest rate volatility lead to fluctuations in bond prices. This reflects the fact that, in practice, these two factors are important determinants of the value of bond portfolios. We introduce PPB to our simulation analysis to assess the expected benefits of this strategy by comparing it to a position in the Bund futures contract.

Denote τ the time until expiration of the option and T the remaining term of the bond at the initial date. We model the position in the Bund futures as a position in the bond that is held from t_0 to $t_0 + \tau$. Hence T-τ is the remaining term of the discount bond (i.e., the duration of the coupon bond) at option expiration. At t_0 the maturity of the bond is equal to T. At $t_0 + \tau$ the bond with remaining maturity T-τ is sold and a new position is taken up in a bond with maturity equal to T, which is held up to $t_0 + 2\tau$ and so forth. This corresponds to a futures position that is held for three months and then rolled over. In our simulations, we take τ = 3 months and T-τ = 7.8017.

We construct the PPB strategy in the following way, which is similar to Merton et al. (1982, p.35). The portfolio held is made up of a number N of the underlying bond plus the same number of put options. We then scale the initial investment to be equal to an amount I, say €100. Put options with time to expiration equal to τ are bought at t_0 so that they expire at $t_0 + \tau$. Hence, an option pricing formula is only needed to establish the premium at the initial investment, not for the payoff at expiry. We choose the strike price *K* of the put option as a function of the price of the bond *F(r,V,T)* at the date when the option is bought.

One outstanding question is the choice of the strike price of these options. Given the fact that exchange-traded options are issued with strike prices rather close to the current price of the underlying asset, an investor will choose from this proposed range of strike prices and end up with options that are not too far in or out of the money. The typical range of moneyness considered in the literature on options strategies is from 10% out of the money to 10% in the money. Hence, Merton, Scholes, and Gladstein (1982) consider moneyness of -10%, 0% and 10%. Likewise, Figlewski, Chidambaran, and Kaplan (1993) consider moneyness of -10%, -5%, 0%, 5% and 10%. As outlined in Merton, Scholes, and Gladstein (1982), there is no single best alternative for the strike price. Instead, the choice depends on investor preferences such as risk tolerance. In what follows, we decide to set the strike price to 10% out of the money in our base case so that K/ F(r,V,T) = 0.9. This corresponds to an investor who is willing to take on some downside risk in any sub-period and is concerned about decreasing profitability of the strategy when the strike price is increased (Macmillan 2000, Chapter 17). Fabozzi (1996, Chapter 16) notes that in the context of bond portfolio management protective puts are usually implemented with out-of-the money puts on bonds or futures.

We generate scenarios for the position in the Bund futures and for the PPB strategy over one year with rebalancing taking place at the beginning of months 1, 4, 7, and 10.

2.1.2. Results

We first conduct the base case experiment using given parameter values, and then perform a variety of robustness checks.

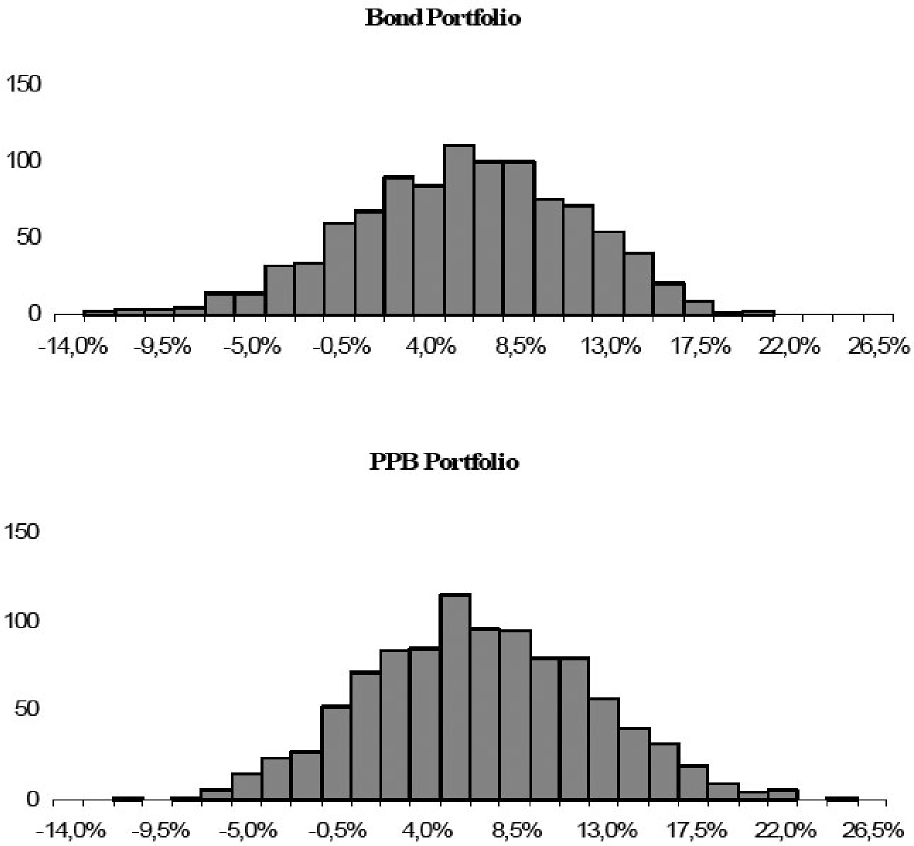
Table 3: Risk and return of a Bund futures strategy and Protective Put Buying Strategy for a one year investment horizon. The left part of the table indicates the percentiles of the return distribution over 1000 scenarios that we generated. The right part of the table indicates some standard performance measures based on this distribution. The information ratio is calculated with respect to the Bund futures strategy. "Bond" denotes the Bund futures strategy. "PPB" denotes the protective put buying strategy.

Performance statistics	Bond	PPB
Mean	4.96%	5.98%
Sharpe-Ratio (2%)	0.52	0.70
VaR (95%)	4.59%	3.20%
Skewness	-0.28	0.14
Information Ratio	0.00	0.41

In order to assess the results for the PPB strategy with Bund futures and options on Bund futures compared to the position in the Bund futures only, we look at the portfolio returns after one year. Table 3 shows both the percentiles of the returns distribution and typical performance statistics. These statistics are based on the distribution of the final portfolio value across the 1,000 scenarios that we generate. This is different from calculating such statistics from a time-series of asset returns, as is done in empirical studies.

An examination of the performance statistics leads to the conclusion that the PPB strategy is largely favourable. In particular, the mean return also increases. This stems from the fact that the put option is exercised in scenarios with strongly negative returns. Consequently, the left tail of the returns distribution is cut off, increasing the mean return. This effect is equally apparent from the greater skewness of the PPB strategy. Figure 1 below shows the return distributions for both the futures strategy ("Bond") and the PPB strategy.

Figure 1: Return distributions for both the futures strategy ("Bond") and the PPB strategy.



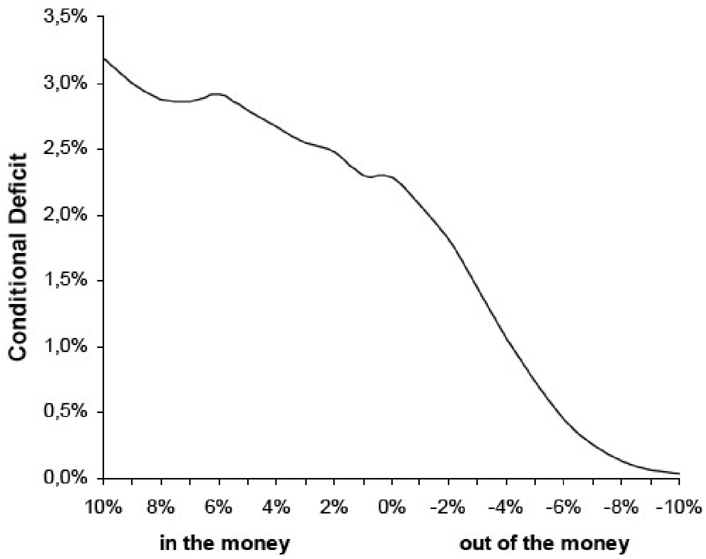
An examination of the probability distribution functions of annual returns confirms the results in table 3. Focusing on negative returns below 7%, it can be seen that PPB has less frequent losses of this magnitude.

It is not true, however, that "PPB" dominates "Bond Only" for each single scenario. For a given scenario, the return of "PPB" could be lower than that of "Bond Only". Intuitively speaking, one may actually expect that the cost of purchasing the downside protection will have an impact on performance. A straightforward calculation actually shows that the probability of underperforming the bond futures strategy is as high as 73.20% when the investor purchases the puts. This results from paying the option price at the beginning of every three-month period. However, the negative effect of paying the option price does not outweigh the benefits of avoiding the most negative drawdowns, as can be seen from the complete returns distribution. In other words, PPB does underperform slightly when bond markets are doing well, and outperforms significantly when bond markets are doing poorly.

Since we are concerned that our results may be driven by our choice of strike price, we vary K and compare PPB to the simple futures strategy for various values of K. From a different standpoint, one can argue that rather than choosing a given value for K, an investor would be interested in choosing the optimal value. This issue is also addressed by our robustness analysis.

Changing the values of K does not lead to a significantly different evaluation of PPB. For a wide range of strike prices, it is favourable when compared to the simple futures strategy. The graph in figure 2 below shows the evolution of the conditional relative deficit, i.e., the average under-performance of PPB across scenarios that are such that PPB out-performs the bond only portfolio, as a function of the degree of moneyness chosen for the put options. (As recalled above, our base case corresponds to a level of moneyness equal to -10%).

Figure 2: Conditional under-performance of the option strategy for different strike prices



The cost of the protection is apparent from this graph. With rising levels of protection (in other words, as the strike for the put option is chosen more and more into the money), underperformance increases. Again, it should be noted that this is the magnitude of underperformance conditional on the fact that PPB underperforms, i.e., given that the put option is not exercised and thus that the returns of PPB are than those where no option is bought. Our choice of protection consistent with -10% moneyness, consistent with that found in related papers, is supported by this analysis, as it leads to a conditional deficit converging to zero.

2.2. Optimal Allocation

The previous simulations considered stand-alone investments in either the options strategy or the bond futures strategy. PPB clearly dominates bond futures. It has a lower downside risk, all while achieving returns that are considerably higher than those for the bond futures strategy. Investors looking for capital preservation would naturally favour such an investment. The assessment of the stand-alone benefits ultimately suggests that an investor should replace his bond portfolio with a suitably designed option strategy.

However, instead of looking at choices between single assets, it is perhaps more relevant to examine the benefits of investing in the option strategy in a portfolio context, as an addition to the bond futures strategy. This subsection turns to this issue.

2.2.1. Description of the Optimization Problem

Benefits of the asymmetric characteristics of options are expected to arise in a framework that explicitly takes into account the asymmetry of the returns distribution, such as in the context of quintiles of the return distribution (such as Value-at-Risk). Since long positions in put options can truncate the returns distribution at a given level, they are expected to constitute a significant portion of the optimal portfolios of investors with downside risk preferences.

Our portfolio choice problem is between the bond futures strategy and the PPB strategy described above. We test two optimisation objectives: our first risk measure is the Value-at-Risk (at 95% confidence) and the second is the variance of portfolio returns. The time horizon is one year, as in the section above.

2.2.2. The Results

The aim of our exercise is to show the risk-return characteristics of minimum risk portfolios with respect to the two risk measures. We also want to analyse the resulting optimal allocation decision, i.e., the weights given to protective put buying and the simple bond futures strategy. It should be kept in mind that PPB itself contains a position in the bond futures and simply adds the long position in the put option.

Table 4 below shows the risk and return characteristics of the minimum risk portfolios. In addition, the last two lines of the table show the improvement over the case where the portfolio consists of 100% invested in the bond futures strategy. There is a reduction of volatility (standard deviation of returns across simulated portfolio wealth at a horizon of one year on our 1,000 paths), but it is not very significant in economic terms. However, the reduction of Value-at-Risk is very pronounced, and this even in the case of the minimum variance portfolio, which reduces the Value-at-Risk by 15%. The Minimum Value-at-Risk Portfolio even reduces the Value-at-Risk by 32%.

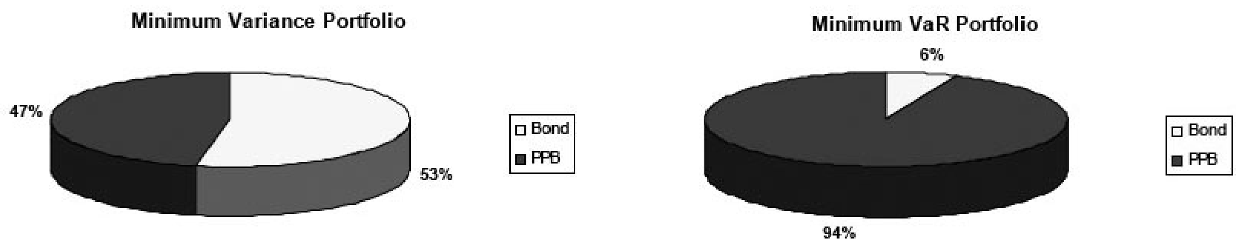
Table 4: Risk and return characteristics of minimum risk portfolios

	Min VaR	Min Variance
Mean	5.92%	5.44%
Std Dev	5.64%	5.52%
VaR (95%)	3.20%	3.90%
Sparpe-Ratio (2%)	0.69	0.62
Skewness	0.11	-0.10
Reduction of Volatility	0.04%	2.21%
Reduction of VaR	30.36%	15.13%

These results clearly show that assessing the value of the option strategy in terms of standard deviation is misleading, since the standard deviation does not fully describe the risk in the case of asymmetric return distributions. The main benefit of including the option strategy, fully apparent only when looking at the Value-at-Risk, which captures the asymmetry of the returns distribution, is that it introduces positive asymmetry to the portfolio.

In addition to assessing the resulting risk-return characteristics, we are interested in what percentage an investor would allocate to the option strategy given that one is averse to normal risk (standard deviation) or extreme risks. The graphs in figure 3 below show that PPB is included with a significant weight in both the minimum variance and the minimum Value-at-Risk portfolio. However, the allocation in the latter case is far more important. This is not surprising given that the optimisation objective in the Value-at-Risk case leads to a preference for the asymmetry implicit in PPB. On the other hand, it appears that even under the volatility objective, the weight attributed to PPB is significant.

Figure 3: Composition of minimum risk portfolios



In summary, option strategies are a useful supplement to a pure linear exposure to fixed-income instruments. It should be noted that rather than designing an optimal options strategy, we define the strategy ex-ante, which suggests that even greater benefits may be obtained by an investor if design is optimal. In other words, the fact that the optimal allocation to PPB is significant even though such payoffs may not necessarily be optimal strongly suggests that the use of derivatives can significantly improve investors' welfare.

In view of the academic research (see in particular Leland 1980 as well as subsequent theoretical studies on the subject), it actually appears that the optimality of a linear payoff is the exception rather than the rule (which of course is obvious from the fact that the set of non-linear payoffs trivially encompasses the linear ones). While this is true in a pure asset management context (such as an investment process of a mutual fund), it is desirable to test whether non-linear payoffs will also be attractive for liability-driven forms of portfolio management. We turn now to an assessment of the strategy used in this section in an asset-liability management context.

3. Managing Liabilities with Interest Rate Derivatives

As recalled in the introduction, the recent pension fund crisis and the emergence of new regulatory and accounting standards have led to greater scrutiny of the risk management practices of institutional investors. A paradigm change is currently taking place in asset management that puts performance in relation to the institution's liabilities at the forefront of the decision-making process. Indeed, a variety of liability-driven investment solutions have emerged in response to the dramatic impact of equity bear markets at the beginning of the new millennium and to the sharp increase in liabilities caused by falling interest rates.

A typical pension plan is exposed to significant interest rate risk emanating from a duration mismatch between assets and liabilities. This exposure is permanent and represents a large, unacknowledged, strategic bet on interest rates and the mismatch in duration exposes the plan to uncompensated risk. Assuming that most pension plan liabilities have an average duration of between 10 and 15 years, it is obvious that using government bonds of a duration significantly lower than that of the liabilities will in an opportunity cost and in great risk/reward differences. A fall in interest rates will have a greater impact on the value of the liabilities than on the value of the assets, with the size of the surplus decreasing as a result. A positive duration gap indicates that, on average, assets are more interest rate sensitive than liabilities. Thus, when interest rates rise (fall), assets will fall proportionately more (less) in value than liabilities and the market value of equity will fall (rise) accordingly. On the other hand, a negative duration gap indicates that weighted liabilities are more interest rate sensitive than assets. Thus, when interest rates rise (fall), assets will fall proportionately less (more) in value than liabilities and the market value of equity will rise (fall).

The most straightforward approach to asset-liability management, focusing explicitly on risk management, is a simple matching strategy, where the investor tries to hold assets that replicate his liabilities. If exact replication of the cash flows is achieved, the assets will perfectly match the liabilities and there is no risk of shortfall, as well as no chance for surplus. Let us assume for example that a pension fund has a commitment to pay out a monthly pension to a retired person. Leaving aside the complexity relating to the uncertain life expectancy of the retiree, the structure of the liabilities is defined simply as a series of cash outflows to be paid, the real value of which is known today. It is possible in theory to construct a portfolio of assets whose future cash flows will be identical to this structure of commitments.

Instead of perfectly matching the cash flow, institutional investors will typically take a duration-matching approach, where the interest rate sensitivity of the assets matches that of the liabilities. While OTC contracts such as inflation and interest-rates swaps can prove useful in the implementation of liability-matching portfolios, we argue in this section that exchange-traded alternatives, such as futures, can be regarded as natural cost-efficient alternatives. In a nutshell, interest rate swaps are a natural fit for cash-flow matching strategies, which involve ensuring a perfect static match between the cash flows from the portfolio of assets and the commitments in the liabilities, while exchange-traded futures are the instruments of choice for immunisation, which allows dynamic management of the residual interest rate risk created by the imperfect match between the assets and liabilities.

In the context of our simulation model, we evaluate the benefits of different derivatives strategies by observing the impact of price fluctuations of fixed-income instruments. To analyse derivatives strategies in asset-liability management, we must add to the paths for asset prices in section two simulated paths for liabilities.

3.1. The Investor's Liabilities

In an attempt to focus on a stylised institutional investor, we model the liabilities as a short position in a global bond index, which can be represented by a zero-coupon bond with constant time-to-maturity. The Longstaff-Schwartz framework allows us to easily price liabilities modelled in this manner. Our way of representing liabilities reflects the preponderant impact of changes in interest rates and in interest rate volatility on the liabilities of institutional investors. As a result, liabilities would be perfectly correlated with returns on a bond index. In practice, there are certainly other factors, such as actuarial uncertainty, that determine the returns on liabilities. It is for this reason that we introduce a disturbance term in our liability process t I , as a convenient reduced-form way to achieve a target correlation between liabilities and the discount bond. In our model, liabilities then correspond to the returns of the discount bond with price L plus a Gaussian white noise:

$$\frac{dl_t}{l_t} = \rho \frac{dL_t}{L_t} + (1-\rho)a_t \qquad a_t \sim N(\mu_l, \sigma_a^2)$$

where a is orthogonal to $\frac{dL_t}{L_t}$, the return process of the bond, and where we take $\rho = 0.8$, $\sigma_a = 0.05$, and we set μ_l to be equal to the mean of L_t.[4]

The duration of the discount bond is chosen to be equal to 10 years in a first case and equal to 15 years in a second case. This choice corresponds to the typical pension plan liability duration of 10 to 15 years (Farley 2003). According to van Dootingh (2004), the average duration of liabilities is approximately 15 years. In the following subsection, we consider the effect on liability-relative performance of investing in different derivatives strategies. In order to isolate the effects of short-term changes in the interest rate and interest rate volatility, we assess the gap between assets and liabilities at a three-month horizon.

3.2. Bund Futures and Options

We will look first at our base case, in which the investor holds simply a position in the Bund future. We assess the outcome of investing 100% of the assets in this strategy in terms of shortfall measures. Since the duration of the Bund future is lower than that of the liabilities we model, it can be expected that the shortfall risk will be significant. A major problem in the management of liability risk is actually that most institutional investors (pension funds, for example) face liabilities that have very long maturities. While in principal the liability position of these investors can be associated with a short position in a bond, the bond market may not offer any instruments of such a long maturity. Therefore, it is difficult to find an appropriate instrument for hedging purposes (see the next subsection for the benefits of using futures available on longer-term bonds). An alternative to managing expected shortfall on the asset side may be to use options on interest rate futures, as in the PPB strategy described earlier in this paper. Because of the convex nature of the payoffs, it is expected that PPB will lead to a reduction in shortfall.

Table 5 shows the percentiles and the mean of the difference between assets and liabilities at the three months horizon.

Table 5: Surplus/Deficit between assets and liabilities after a three month period for the Bund futures strategy and Protective Put Buying Strategy. The percentiles of the surplus/deficit distribution over 1000 scenarios that we generated are shown. The results are for liabilities with 15 year duration.

	Bund	PPB
Minimum	-0.77%	-0.78%
5%	-0.56%	-0.56%
25%	-0.16%	-0.17%
Median	0.09%	0.08%
Mean	0.34%	0.70%
75%	0.33%	0.32%
95%	0.74%	0.76%
Max	1.14%	6.97%

Unsurprisingly, the shortfall risk is significant, as can be seen from the fact that with 25% probability, the assets will be lower than the liabilities both for the simple futures strategy and for PPB. Somewhat more surprisingly, PPB does not reduce the shortfall risk, but actually increases significantly the performance of the portfolio. This latter statement can be verified when looking at the mean surplus and the maximum surplus. When thinking about the impact of an isolated shock to interest rates, these results become more intuitive.

Consider a steep rise in interest rates. This will lead to a decline in the value of the Bund futures and to an even steeper decline in the value of the liabilities, on account of their longer duration. Therefore, we will observe a surplus after the interest rate shock for the Bund futures strategy. For PPB, the surplus will be even more positive. This comes from the fact that the decline in value of the PPB portfolio will be limited due to the protection from

4 - It should be noted that the return on liabilities is usually considered to be higher than that of a bond with similar duration. Since the difference in performance is typically compensated by additional contributions, which we do not -model here, we have chosen to abstract way from this added complexity in this paper.

the put options. Next, consider a steep decline in interest rates. The value of the Bund futures strategy will rise. Liabilities will rise even more in value, so that we will observe a shortfall for the Bund futures strategy after the interest rate rise. If the investor holds the PPB strategy, the value of his/her assets will also rise, but less than for the simple futures strategy. The difference is given by the option premium that the investor has to pay.

In short, PPB actually serves as a return enhancer, while being exposed to roughly the same risk of drawdown relative to liabilities upon a negative interest rate shock (decline in the interest rate). While it may be interesting to look at alternative option strategies in the context of asset-liability management, the most straightforward way to manage the shortfall risk is to pursue simple duration-matching strategies.

3.3. Introducing the Buxl Contract

As highlighted above, having long-term maturity instruments for hedging purposes at their disposal is a critical need for institutional investors since the convexity of the price-yield relation for a bond increases with longer maturities. The significant shortfall risk for the two strategies based on the Bund futures actually stems from the lower duration compared to the liabilities. This problem is all the more significant in times of low interest rate coupons which also have a positive impact on convexity. Given these problems, we propose to assess the usefulness of a 30-year bond future for hedging purposes. This contract corresponds to a duration of roughly 18 years, which is actually higher than the duration for the liabilities in our model (10 years and 15 years), while the duration of the Bund contract is lower in both cases.

Figure 4 below shows the distribution of the difference between assets and liabilities at the three-month horizon. Again, this allows us to assess the impact of interest rate shocks on the investor's situation, given that he/she faces liability constraints. The histograms on the left show the surplus/deficit distribution when liabilities correspond exactly to a short position in a bond index. The histograms on the right show the surplus/deficit distribution when the liabilities have a correlation of only 0.8 with the bond index, i.e., when we have introduced a white noise disturbance. The upper graphs show the investor's holding the simple Bund futures strategy, the middle graphs correspond to the Buxl futures strategy, and the lower graphs to investor attempts to match the duration of liabilities by mixing both strategies. It can be seen that the Buxl futures strategy leads to a lower variability of the distribution, while the duration-matching strategy achieves the lowest distribution. In addition, it appears that even if the liabilities deviate significantly from a zero-coupon bond, the duration-matching technique is useful, i.e., the main risk stems from the interest rate changes.

We obtain the weights of the Bund and Buxl strategy by minimising the shortfall variance. We favour this approach to simple duration calculations, which rely on certain assumptions, notably that the yield curve is affected only by small parallel shifts, which may not hold in our setup.

Figure 4: Surplus/Deficit between assets and liabilities after a three month period. 10 year duration for liabilities

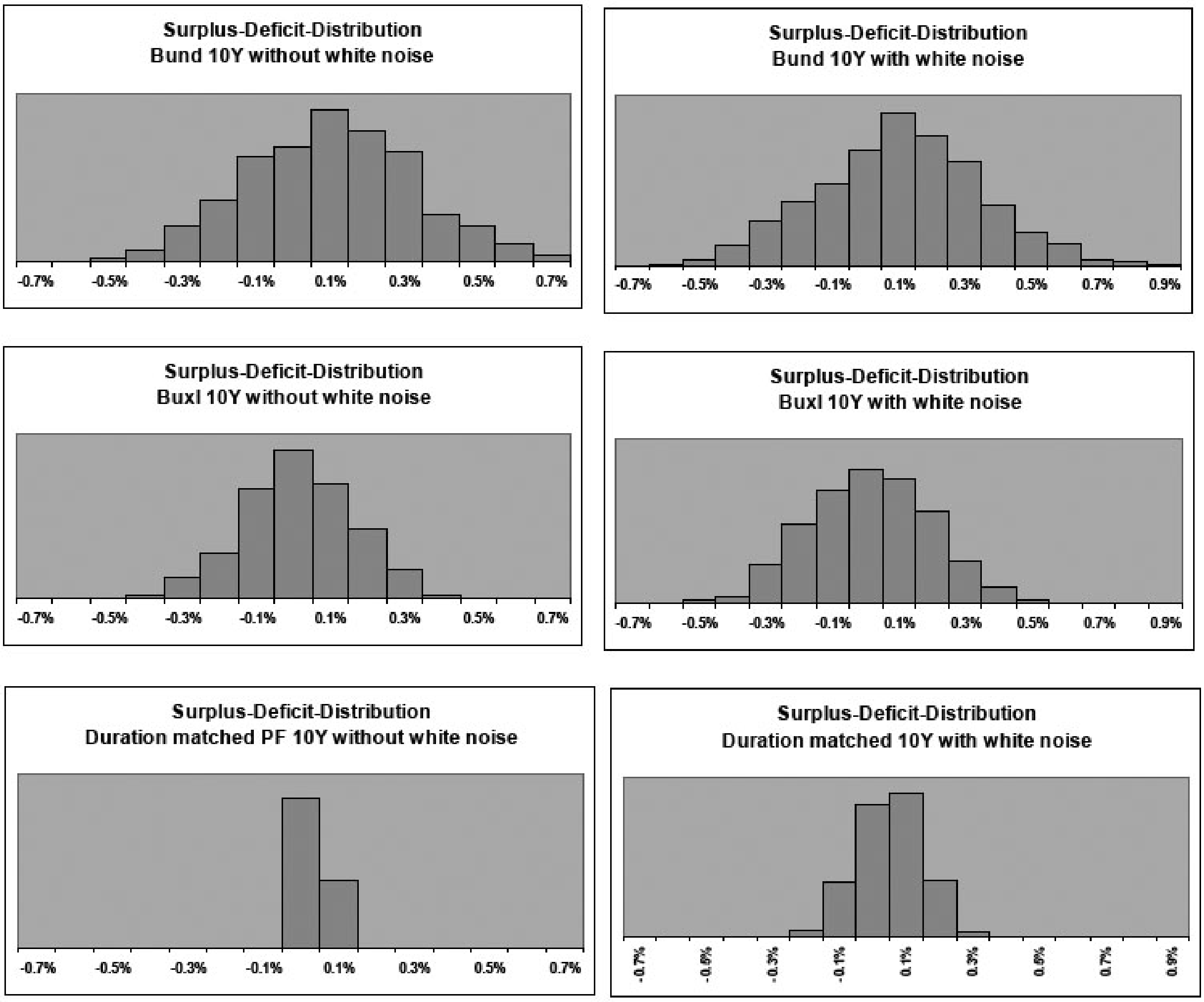


Figure 5 shows the results when liabilities have a duration of 15 years. The Buxl futures allows an even more significant reduction, which is not surprising given that its duration is close to those for the liabilities.

Figure 5: Surplus/Deficit between assets and liabilities after a three month period. 15 year duration for liabilities

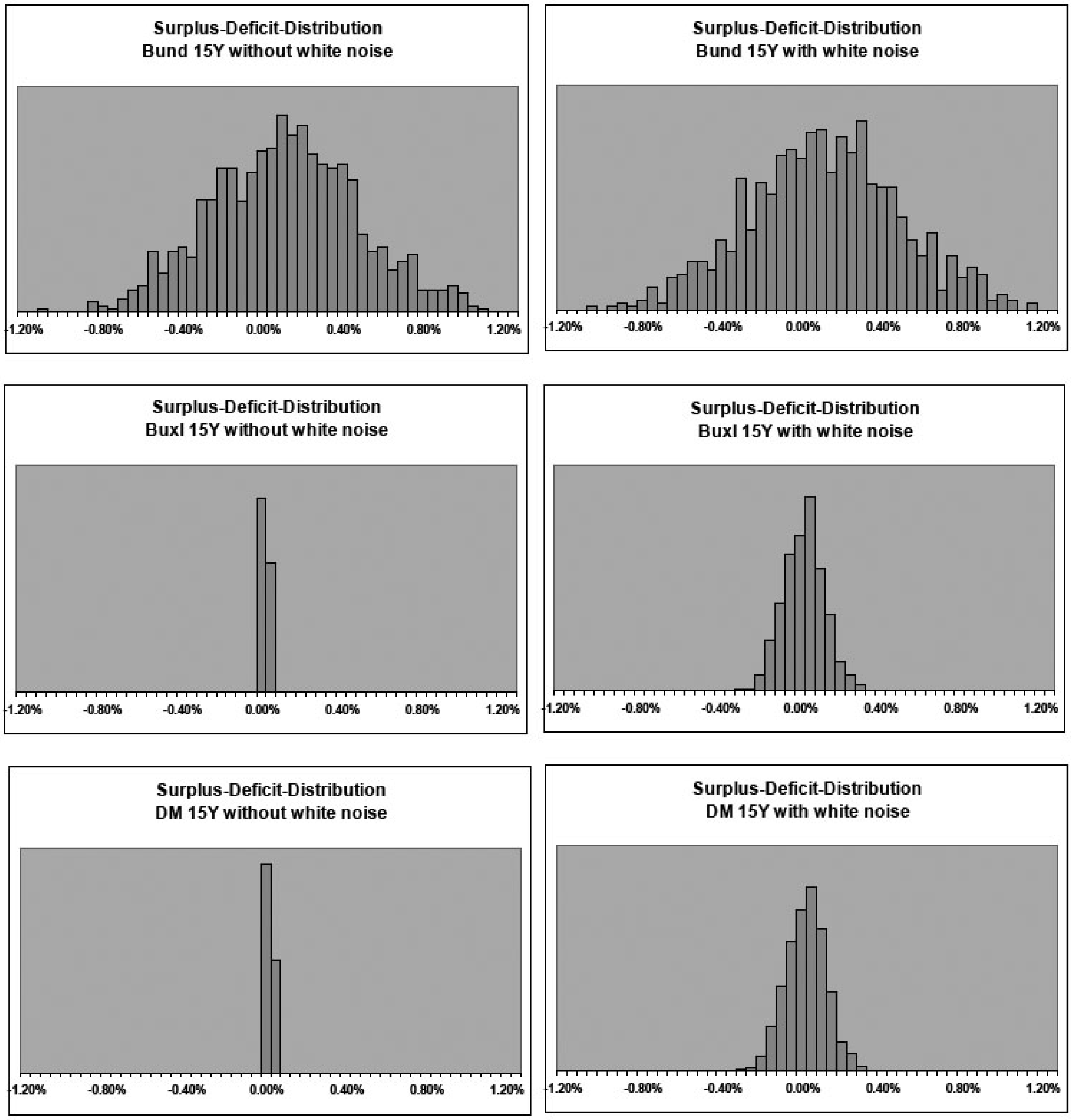


Table 6 shows the weights that we obtain for the different cases. The positive contribution of the Buxl future is obvious. The longer duration of these futures contracts leads to significant reduction of the surplus/deficit variance, allowing investors with liability constraints to achieve matching superior to that achieved when the Buxl futures is unavailable. This is reflected in the significant allocations the Buxl futures strategy obtains in the duration-matching portfolios, which range from 60 to 98.8%.

Table 6: Composition of duration matching portfolios

	Duration of Liabilities	Bund	Buxl
Without	10Y 3	9.7%	60.3%
white noise	15Y	3.8%	96.2%
With	10Y	39.2%	60.8%
white noise	15Y	1.2%	98.8

A comment on the limits of our ALM risk management analysis is in order. In practice, simultaneous management of interest rate risk and other risk factors is known to be extremely difficult (Le Vallois et al 2003). This is captured in our model by the error term which reflects risk unrelated to interest rates. As expected, the results with the white noise perturbation are less favourable than those without the white noise perturbation.

Another, probably greater, disadvantage of the risk minimisation strategy considered here (as well as its more standard and specific applications such as cash-flow matching strategies, or immunisation strategies) is that it represents an extreme position not necessarily optimal for the investor in the risk/return space. In fact, the surplus risk minimisation approach in asset-liability management is the equivalent of investing in a low risk asset in an asset management context. It allows tight management of risk, but the absence of return, a direct consequence of the absence of risk premia, makes this approach very costly and leads to an unattractive level of contribution to the assets. PPB, described above, may be a partial solution to this problem.

Conclusion

In this paper, we assess the value of using bond futures and options on these futures. We examine the value of introducing derivatives both in a pure asset management context and in an asset-liability management context. We conduct a simulation study based on the Longstaff-Schwartz (1992) interest rate model by generating stochastic scenarios for instruments that correspond to Eurex contracts on long-term German government bonds. In particular, we look at the Bund futures and Buxl futures contracts, as well as options on the Bund futures contract. Our results show that the non-linear character of the returns on put option buying strategies offers appealing risk reduction properties in the pure asset management context. Consequently, such strategies obtain a high weighting in an optimal portfolio, especially when investors are concerned with minimising the extreme risks. In an asset-liability management context, we show that such strategies offer surplus benefits. In order to achieve significant shortfall reduction, the investor typically needs to invest in very long-term contracts that reflect the long duration of most liability constraints. Our results underline the importance of such contracts for asset-liability management.

Future research may adopt a similar perspective to assess the value of options strategies. In this paper we have limited the investor's asset menu to derivatives strategies that are defined ex-ante. An extremely relevant research topic, both from a practical and from a theoretical point of view, is optimal product design, given the preferences of asset managers or institutional investors.

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