A Pitfall with DSGE–Based, Estimated, Government Spending Multipliers

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Abstract

This paper examines issues related to the estimation of the long-run government spending multiplier (GSM) in a Dynamic Stochastic General Equilibrium context. We stress a potential source of bias in the GSM arising from the combination of Edgeworth complementarity between private consumption and government expenditures and countercyclical government expenditures. Due to cross-equation restrictions, omitting the endogenous component of government policy at the estimation stage would lead an econometrician to underestimate the degree of Edgeworth complementarity and, consequently, the long-run GSM. An estimated version of our model with US postwar data shows that this bias matters quantitatively.

Keywords: DSGE models, Edgeworth complementarity, Government spending rules, Maximum likelihood.

JEL Class.: C32, E32, E62.

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1. Introduction
In the current crisis context, there has been a renewed academic and policy interest in studying the effects of government activity.¹ A key quantity that has attracted considerable attention is the government spending multiplier (GSM), i.e. the increase in output consecutive to an increase in government spending.

In this paper, we study issues related to the estimation of this multiplier in a Dynamic Stochastic General Equilibrium (DSGE) context. We stress a potential source of bias in the GSM arising from the combination of (i) Edgeworth complementarity between private consumption and government expenditures and (ii) countercyclical government expenditures. We find that the degree of Edgeworth complementarity and the cyclicality of policy interact at the estimation stage through cross-equation restrictions, paving the way for potential biases. At the same time, we show that the GSM is an increasing function of the degree of Edgeworth complementarity between private consumption and government expenditures. Thus, any bias in the degree of Edgeworth complementarity translates into a biased GSM. The underlying mechanics are the following.

Several recent papers have documented that government spending policy is countercyclical, see, e.g., John Bailey Jones (2002), Eric M. Leeper, Michael Plante & Nora Traum (2010).² This raises a severe challenge for Neoclassical models. In those setups, following any shock such that both output and consumption decline, countercyclical policy triggers an increase in public spending. This in turn increases output but reduces consumption even more (crowding-out effect), finally making private consumption even more negatively correlated with public expenditures than under an exogenous spending policy. This seems to be at odds with post-war US data. Typically, over the sample used in this paper, we observe a correlation between the growth rates of these aggregate quantities around 0.24. Allowing for Edgeworth complementarity helps mitigate this problem. With such a mechanism, a rise in public expenditures would make people want to consume more, thus counteracting the crowding-out effect. As a consequence, given a certain unconditional correlation between private consumption and government expenditures that we seek to match, allowing for a very countercyclical policy will require a high degree of Edgeworth complementarity. This will mechanically translate into a large GSM. Conversely, omitting countercyclical policy will imply a small degree of complementarity, thus yielding a downward-biased GSM.

To establish these results formally, we first work out a simple model with only limited dynamic features. The model is simple enough to provide us with an analytical characterisation of the bias that would arise from omitting a countercyclical component to government spending policy. We use this simple framework to identify configurations in which this bias would be likely. We show that in this simple setup, omitting the countercyclical policy rule at the estimation stage would always yield a downward-biased estimate of the GSM, provided shocks to government expenditures are not the only perturbations affecting the economy. Because countercyclical policy and Edgeworth complementarity work in opposite directions in terms of generating a certain pattern of correlation between consumption and government expenditures, we can reinterpret this bias as a simultaneous equation bias. As a matter of fact, the simple model allows us to derive a formula for the bias that closely resembles those appearing in standard econometrics textbooks in a demand–supply framework. By analogy with this celebrated framework, an econometrician omitting the countercyclical spending rule risks recovering the policy rule parameter when trying to estimate the private response to public spending. In all likelihood, this will happen when shocks to government spending account for a small portion of fluctuations and/or the feedback effect in the policy rule is strong.

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¹ - See, among others, John F. Cogan, Tobias Cwik, John B. Taylor & Volker Wieland (2010); Lawrence J. Christiano, Martin Eichenbaum & Sergio Rebelo (2011), Jesus Fernandez-Villaverde (2010), Harald Uhlig (2004). The common theme of these papers is to investigate under which circumstances the multiplier may or may not be large.
² - In closely related papers, Vasco Cúrdia & Ricardo Reis (2010) and Ellen R. McGrattan (1994), the forcing variables are assumed to follow vector autoregressive processes, which can be interpreted as reduced-form policy rules when it comes to exogenous policy variables such as government spending or taxes. Importantly, in spite of specification or sample differences, these papers all find significant countercyclical policy rules that prove essential to the models fit.
In a second step, using post-war US data, we estimate a quantitative model version via maximum likelihood techniques. We show that the same sort of bias is present when the econometrician omits the countercyclical component of government policy. This, in turn, translates into significant differences in the estimated long-run government spending multiplier. In our benchmark specification with Edgeworth complementarity and countercyclical policy, the implied long-run multiplier amounts to 1.31. Using the same model and imposing an exogenous policy rule, we obtain a multiplier of 0.97, significantly smaller than our benchmark value by 0.34 point. Such a difference is clearly not neutral if the model is used to assess recovery plans of the same size as those recently enacted in the US. To illustrate this more concretely, we feed the American Recovery and Reinvestment Act (ARRA) fiscal stimulus package into our model. We obtain that omitting the countercyclical policy rule at the estimation stage would lead an analyst to underestimate the cumulated output effect of this package by 15% of the package itself or, equivalently, around 1.4% of US GDP prior to the shock.3 Clearly, these are not negligible figures. Interestingly, simulating our preferred model while imposing the counterfactual hypothesis that policy is exogenous would not produce very different dynamics. This illustrates that, while countercyclical policy does not seem to play a major role when simulating the model, it turns out to be an essential feature when estimating the model.4

Our estimations yield a GSM which exceeds unity and a near zero multiplier for private consumption. These findings are broadly in the range of values reported by Robert E. Hall (2009) – typically in between 0.5 and 1.7 for output. This range of values derives from various methods. These include, for example, GSM estimates from DSGE models (e.g. Christiano, Eichenbaum & Rebelo 2011, Tommaso Monacelli, Roberto Perotti & Antonella Trigari 2010, Sarah Zubairy 2010), single regressions on government purchases (Robert J. Barro & Charles J. Redlick 2009, Hall 2009), and Structural Vector Autoregressions (SVARs) (e.g. Olivier Blanchard & Roberto Perotti 2002, Dario Caldara & Christophe Kamps 2008, Jonas D.M. Fisher & Ryan Peters 2010).5 Interestingly, the DSGE literature itself generates such a range of values for the GSM: the lower bound obtains in typical calibrated neoclassical setups; the upper bound is obtained in Christiano, Eichenbaum & Rebelo (2011). A key contribution of the latter is to show under which circumstances the multiplier can be much larger than one, typically when the economy has reached the zero lower bound on the nominal interest rate. Our paper adds to this literature by spotting possible estimation biases that naturally emerge from such models. The strength of these models lies precisely in their cross-equation restrictions which significantly contribute to highlight identification problems for the GSM. Our analytical results also point out potential sources of bias in SVARs. This can arise because SVARs are not immune to the omitted policy problem emphasised in this paper. In particular, Dario Caldara (2011) has shown how the calibration of the impact response of the policy variable to an innovation in output can deeply affect the estimated response to fiscal variables, both from actual and simulated data. Our simple setup offers an immediate interpretation of his result. Again, by imposing that government spending does not react on impact to the state of the economy, a recursive identification scheme in the VAR model would lead a researcher to under-estimate the GSM. As in the DSGE setup, the reason why is intuitive. By wrongly assuming an exogenous policy, the stabilising role of government policy is omitted and this contaminates the estimated response of output. When there is no feedback rule or when the contribution of the policy shock is large relative to the non-policy shock, the estimated responses from SVARs recover the true ones. Conversely, when the policy is very reactive or the policy shock accounts for a small portion of fluctuations, SVARs yield severely downward-biased estimates of the GSM.

The rest of the paper is organised as follows. In section 2, we expound the simple model and illustrate the trade-off between Edgeworth complementarity and countercyclical policy in

3. In this paper, we insist on the long-run GSM. The main reason for this is that this number does not directly depend on the persistence of government policy shocks. As shown by S. Rao Ashagi, Lawrence J. Christiano & Martin Eichenbaum (1992), short-run multipliers can prove very sensitive to the persistence of these shocks, which would complicate the comparison between different model versions. Focusing on the long-run GSM allows us to sidestep this problem. However, all our results hold in an assessment of shorter-term multipliers.

4. To complement these results, we conduct several robustness analyses reported in appendix D. First, using our preferred model version as a data generating process, we perform several simulation exercises and obtain quantitative results that echo our analytical formula. Second, we modify the model specification allowing for (i) habits in consumption, (ii) dynamic adjustment costs, (iii) alternative policy rules, and (iv) news shocks in the policy rule. We also investigate the robustness of our results to subsamples. None of our conclusions is affected by these perturbations to the benchmark setup.

5. Notice also that other original identification strategies have been recently used; see e.g. Dario Caldara (2011), Juan-Carlos Suárez Serrato & Philippe Wingender (2011).
terms of matching the observed correlation between output and government expenditures. We then characterize the bias that would result from omitting countercyclical policy. Section 3 develops a quantitative version of this model that we take to post–war US data. We then explore the quantitative implications of policy rule mis-specification. In appendix D, we investigate the robustness of our results, using both actual and simulated data. The last section briefly concludes.

2. A Simple Illustrative Example

In this section, we work out an equilibrium model simple enough to obtain closed–form formulas illustrating how the long–run government spending multiplier is biased when the econometrician omits the endogenous component of public policy.

2.1 The Model

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household seeks to maximize

\[ \max E_t \sum_{i=t}^{\infty} \beta^i \left\{ \log (c_{t+i} + \alpha_g g_{t+i}) - \frac{\eta}{1 + \nu} n_{t+i}^{1 + \nu} \right\} \]  

subject to the sequence of budget constraints \((t \geq 0)\)

\[ c_t \leq w_t n_t - T_t \]

where \(E_t \{ \cdot \}\) is the expectation operator, conditioned on information available as of time \(t\), \(\beta \in (0, 1)\) is the subjective discount factor, \(c_t\) is private consumption, \(g_t\) denotes public expenditures, \(n_t\) is the labour supply, \(w_t\) is the real wage rate, and \(T_t\) denotes lump–sum taxes. The Frisch elasticity of labour supply is \(1/\nu\) and \(\eta > 0\) is a scale parameter.

The parameter \(\alpha_g\), in turn, accounts for the complementarity/substitutability between private consumption \(c_t\) and public spending \(g_t\). If \(\alpha_g \geq 0\), government spending substitutes for private consumption, with perfect substitution if \(\alpha_g = 1\), as in Christiano & Eichenbaum (1992). In this case, a permanent increase in government spending has no effect on output and hours but reduces private consumption, through a perfect crowding–out effect. In the special case \(\alpha_g = 0\), we recover the standard business cycle model, with government spending operating through a negative income effect on labour supply (see Aiyagari, Christiano & Eichenbaum 1992, Marianne Baxter & Robert G King 1993). When the parameter \(\alpha_g < 0\), government spending complements private consumption. Then, it can be the case (depending on the labor supply elasticity) that private consumption will react positively to an unexpected increase in government spending.

The representative firm produces a homogeneous final good \(y_t\) using labour as the sole input, according to the constant returns–to–scale technology

\[ y_t = e^{z_t} n_t \]

Here, \(z_t\) is a shock to total factor productivity, assumed to be iid with \(z_t \sim N(0, \sigma_z^2)\). Profit maximization implies that the marginal productivity of labour equals the real wage, i.e. \(w_t = e^{z_t}\). Government purchases are entirely financed by taxes,

\[ T_t = g_t \]

As in the recent literature emphasising the relevance of stabilising government spending rules (see Leeper, Plante & Traum 2010), among others, we specify a feedback rule of the following form

\[ g_t = \bar{g} \left( \frac{y_t}{y_t - 1} \right)^{-\varphi_g} e^{u_t} \]
where $\bar{g}$ is a scale factor that pins down the deterministic steady–state level of government expenditures and $\varphi_g \geq 0$. This restriction imposes that government spending stabilise aggregate activity, i.e. expenditures increase when output growth is below its average value. With this restriction we make sure that the reduced–form model displays second–order stationarity. The random term $u_t$ represents the discretionary part of policy and is assumed to be iid with $u_t \sim N(0, \sigma_u^2)$.

Finally, the market clearing condition on the goods market writes $y_t = c_t + g_t$.

Combining the household’s first order condition on labour, the profit maximization condition, and the resource constraint, one finally arrives at the equilibrium condition

$$\eta y_t^e = \frac{e^{(1+\nu)z_t}}{y_t - (1 - \alpha_g)g_t}. \quad (4)$$

Conditions (3) and (4) together constitute the equilibrium system governing the dynamics of the above economy. To ensure positiveness of the marginal utility of consumption, we henceforth impose the restriction $\alpha_g > (s_g - 1) = s_g$, where $s_g \equiv \bar{g}/\bar{y} \in (0, 1)$ is the steady–state public spending–output ratio.

In this economy, the long–run GSM is defined as follows.

**Definition 1.** The long–run government spending multiplier, denoted by $\Delta y = \Delta g$, is the increase in steady–state output $y$ after an increase in steady–state government spending expenditures $\bar{g}$, i.e. formally

$$\frac{\Delta y}{\Delta g} = \frac{d\bar{y}}{d\bar{g}}.$$

From this definition and the structure of the above model economy, the following proposition states key properties of the long–run GSM.

**Proposition 1.** Under the preceding assumptions:

1. The long–run government spending multiplier $\Delta y = \Delta g$ is

$$\frac{\Delta y}{\Delta g} = \frac{1 - \alpha_g}{1 + \nu[1 - s_g(1 - \alpha_g)]}.$$

2. The multiplier is a decreasing function of $\alpha_g$.

**Proof.** See Appendix A.

This proposition establishes that the long–run GSM depends on the share of government spending in output ($s_g$), on the inverse Frisch elasticity of labor($\nu$), and on the parameter governing the degree of Edgeworth complementarity between private consumption and government expenditures ($\alpha_g$). Importantly, $\Delta y / \Delta g$ does not depend directly on the degree of countercyclicality of the government spending rule ($\varphi_g$). The main thesis of this paper, though, is that $\varphi_g$ can contaminate the long–run GSM indirectly.

To gain intuition as to how this can happen, we start by loglinearizing the system (3)–(4) in the neighborhood of the deterministic steady state.
This yields
\[ \hat{y}_t = \alpha \hat{g}_t + \zeta z_t \] (5)
\[ \hat{g}_t = -\varphi_g (\hat{y}_t - \hat{y}_{t-1}) + u_t \] (6)

where a letter with a hat denotes the logdeviation (with respect to steady–state value) of the associated variable and the composite parameters \( \alpha \) and \( \zeta \) are defined as
\[
\alpha = \frac{s_g(1 - \alpha_g)}{1 + \nu s_g(1 - \alpha_g)}, \\
\zeta = \frac{(1 + \nu)(1 - s_g(1 - \alpha_g))}{1 + \nu s_g(1 - \alpha_g)}.
\]

In the remainder, to simplify the algebra, we drop the coefficient \( \zeta \) from the dynamic system. This can always be done by rescaling appropriately the standard error of \( z_t \).

For \( \nu \) and \( s_g \) set at given values, the value of \( \alpha \) summarizes the complementarity/substitutability between private and public consumption. This composite parameter and the long–run government spending multiplier are tightly linked since
\[ \frac{\Delta y}{\Delta g} = \frac{\alpha}{s_g} \]

The system (5)–(6) makes clear how the degree of countercyclicality of the government spending rule and the degree of Edgeworth complementarity between private consumption and government spending work in opposite directions in terms of generating a positive correlation between \( y \) and \( g \). Intuitively, Edgeworth complementarity between private consumption and government spending (i.e. \( \alpha_g < 0 \)) tends to increase the correlation between \( y \) and \( g \), since under such a configuration an increase in government expenditures would induce people to consume more; at the same time, a countercyclical policy rule reduces this correlation.

This yields a trade–off: given an observed correlation between output and government spending, a highly countercyclical policy must be compensated by a high degree of Edgeworth complementarity. Conversely, if policy is exogenous, a lower degree of Edgeworth complementarity will suffice to match the observed pattern of correlation between output and government expenditures. This is illustrated in figure 2.1.

This figure shows two iso–correlation loci in the \((\phi_g, \alpha_g)\) plane, depending on the relative sizes of the structural disturbances. Each point in these loci gives a particular \((\phi_g, \alpha_g)\) combination resulting in the same correlation between output and government spending. When \( \sigma_z > \sigma_u \), the iso–correlation locus is decreasing with \( \phi_g \) with a steep slope. This means that, as the degree of countercyclicality on policy increases, it takes more and more complementarity to match the observed correlation. When \( \sigma_u > \sigma_z \), the iso–correlation curve is much flatter but the trade–off still exists.

This trade–off paves the way for a potential bias in the estimated degree of Edgeworth complementarity (and, by virtue of the above proposition, in the estimated multiplier). Suppose that an econometrician seeks to estimate \( \alpha \) but uses a mis-specified model in which \( \phi_g \) is set to zero while actually \( \phi_g > 0 \). The above reasoning suggests that this would result in a downward–biased estimate of \( \alpha \), immediately translating into a downward–biased estimated multiplier. The next section formally establishes this.
2.2 The Effect of Omitting Endogenous Policy

Direct calculations yield the model’s reduced-form

\[ \hat{\dot{y}}_t = \frac{\alpha \varphi_g}{1 + \alpha \varphi_g} \hat{y}_{t-1} + \frac{\alpha}{1 + \alpha \varphi_g} u_t + \frac{1}{1 + \alpha \varphi_g} z_t \]

(7)

\[ \hat{\dot{y}}_t = \frac{\varphi_g}{1 + \alpha \varphi_g} \hat{y}_{t-1} + \frac{1}{1 + \alpha \varphi_g} u_t - \frac{\varphi_g}{1 + \alpha \varphi_g} z_t \]

(8)

From this reduced form, the structural parameters \((\alpha, \varphi_g, \sigma_u, \sigma_z)\) can be recovered using the \(p\) lim of the maximum likelihood estimation or an instrumental variable technique (with a relevant choice of instrumental variables). An easy way to obtain a consistent estimator of \(\alpha\) relies on indirect estimation using the following representation of the reduced form

\[ \hat{\dot{y}}_t = \pi_1 \hat{y}_{t-1} + \epsilon_{1,t} \]

(9)

\[ \hat{\dot{y}}_t = \pi_2 \hat{y}_{t-1} + \epsilon_{2,t} \]

(10)

The \(p\) lim estimators of Section 01 and Section 02 are given by

\[ \hat{\pi}_1 = \frac{E\{\hat{y}_t \hat{y}_{t-1}\}}{E\{\hat{y}_t^2\}} \text{ and } \hat{\pi}_2 = \frac{E\{\hat{\dot{y}}_t \hat{y}_{t-1}\}}{E\{\hat{\dot{y}}_t^2\}} \]

from which we deduce

\[ \hat{\alpha} = \frac{\hat{\pi}_1}{\hat{\pi}_2} = \frac{E\{\hat{y}_t \hat{y}_{t-1}\}}{E\{\hat{\dot{y}}_t \hat{y}_{t-1}\}} \]

From (7)–(8), we obtain:

\[ E\{\hat{y}_t \hat{y}_{t-1}\} = \frac{\alpha \varphi_g}{1 + \alpha \varphi_g} E\{\hat{y}_t^2\} \text{ and } E\{\hat{\dot{y}}_t \hat{y}_{t-1}\} = \frac{\varphi_g}{1 + \alpha \varphi_g} E\{\hat{\dot{y}}_t^2\} \]
The indirect estimator $\hat{\alpha}$ of $\alpha$ is thus consistent. Similarly, $\hat{\varphi}_g$ is also a consistent estimator of $\varphi_g$. Now, imagine the econometrician ignores the feedback rule and seeks to estimate the parameter $\alpha$ from data $(\hat{y}_t, \hat{z}_t)$ generated by the model (7)–(8). The model considered by this econometrician is thus of the form

$$\hat{y}_t = \hat{\alpha} \hat{u}_t + \hat{z}_t$$  (11)

$$\hat{y}_t = \hat{u}_t.$$  (12)

By ignoring the parameter $\varphi_g$, the econometrician is implicitly estimating the government spending effects on output through a single-equation approach in a simultaneous-equation setup. As is well known from standard econometrics textbooks, she potentially faces a severe *simultaneous-equation* bias (see William H. Greene 1997, James D. Hamilton 1994).

To see this clearly, the ML estimator $\hat{\alpha}$ of $\alpha$ would be

$$\hat{\alpha} = \frac{E\{\hat{y}_t \hat{u}_t\}}{E\{\hat{y}_t^2\}}$$

which simply corresponds to the OLS estimator. While $\varphi_g$ exerts no influence on the long-run multiplier (see proposition 1), the next proposition establishes that this parameter corrupts the estimated composite parameter $\hat{\alpha}$ when policy is assumed to be exogenous.

**Proposition 2.** Under the previous hypotheses

1. The Maximum Likelihood estimator $\hat{\alpha}$ of $\alpha$ is

$$\hat{\alpha} = \frac{\alpha(1 + \alpha \varphi_g)\sigma_u^2 - \varphi_g \sigma^2_z}{(1 + \alpha \varphi_g)\sigma_u^2 + 2\varphi_g^2 \sigma^2_z}. \tag{13}$$

2. Whenever $\sigma_Z > 0$ and $\varphi_g > 0$, $\hat{\alpha}$ is downward-biased.

3. $\forall \sigma_Z > 0$ and $\forall \sigma_u > 0$, we have

(a) If $\alpha \geq \sigma_Z / \sigma_u$, then

$$\frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) < 0, \quad \forall \varphi_g \geq 0.$$

(b) If $\alpha < \sigma_z / \sigma_u$, $\exists \varphi_g(\alpha, \sigma_z, \sigma_u) > 0$ such that

$$\frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) \leq 0, \quad \forall \varphi_g \in [0, \varphi_g(\alpha, \sigma_u, \sigma_z)],$$

$$\frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) > 0, \quad \forall \varphi_g > \varphi_g(\alpha, \sigma_u, \sigma_z).$$

**Proof.** See Appendix B.

Part 1 of proposition 2 establishes that the estimated value of $\hat{\alpha}$ is thus corrupted, in a non-linear way, by $\varphi_g$, $\sigma_u$ and $\sigma_Z$. The OLS regression does not pin down the effects of $\hat{y}$ on $\hat{y}$ but an average of private behaviour and public policy, with weights that depend on the relative size of the shocks’ variances. Notice that this is more or less the formula displayed in standard econometrics textbooks in a demand-supply setup (e.g., see Hamilton 1994, chap. 9).

Part 2 of proposition 2 indicates under which circumstances omitting the endogenous component of government spending policy would result in a downward-biased estimate of $\alpha$. Notice that
under such circumstances, the long–run GSM is systematically downward–biased, by virtue of Proposition 1. Conversely, the only circumstances in which the bias vanishes are either $\varphi_g = 0$ or $\sigma_u = 0$. When $\varphi_g = 0$, the bias is obviously zero since, in this case, the model is well specified. If $\sigma_z = 0$, the bulk of fluctuations in $\hat{y}$ are accounted for by government spending shocks. In this case, the endogeneity bias vanishes. Endogenous public spending is positively related to the shock $u$ and the (inverse) government policy shifts along the output equation.

When $\sigma_z > 0$ and $\sigma_u > 0$, the bias may increase or decrease with $\varphi_g$, depending on $\alpha$, $\sigma_z$, and $\sigma_u$, as stated in part 3 of Proposition 2. If $\alpha \geq \sigma_z / \sigma_u$, the bias increases. If $\alpha < \sigma_z / \sigma_u$, the bias increases with $\varphi_g$, up to a threshold value above which it decreases. However, the bias never reverts back to zero since $\lim \varphi_g \rightarrow \infty = 0 < \alpha$.

In the special case $\sigma_z > 0$ and $\sigma_u \rightarrow 0$, government spending shocks do not contribute much to the variance of $\hat{y}$. In this case, we obtain $\hat{\alpha} \rightarrow -1/(2\varphi_g)$. Thus, omitting the endogeneity of government spending would lead us to estimate a negative value of $\hat{\alpha}$. This is because endogenous public spending is negatively related to the shock $z$ which shifts aggregate output. The covariance between $\hat{y}_t$ and $\hat{y}_t$ is negative and thus the estimated effect of public spending on output is negative. We are in the case when the output equation moves along the policy rule equation (which is truly downward slopping). In this case, the econometrician almost recovers the reverse government policy rule.

To sum up, we have shown analytically in a tractable model that omitting the endogenous component of government spending can result in a downward–biased estimate of the long–run government spending multiplier. In this simple setup, the downward bias is a mix of a simultaneous–equation bias and an omitted–variable bias. It is the result of two conflicting economic forces, one that magnifies the correlation between output and government spending (Edgeworth complementarity) and the other that reduces it (countercyclical government spending rule). In the following section, we consider a quantitative DSGE model which we estimate on US data via maximum likelihood techniques. While the model is too complicated to get such a sharp bias characterization, it proves a useful tool to investigate whether omitting the endogenous component of government spending actually results in a quantitatively significant bias in estimated government spending multipliers.

3. A Quantitative Model

We now work out a quantitative extension of the previous model that we formally take to the data. We extend the previous setup by allowing for capital accumulation, habit formation in leisure decisions, and multiple shocks. While the model is arguably very stylized, it turns out to deliver a good fit. For simplicity, the model abstracts from Keynesian features such as sticky prices or wages. Such mechanisms, however, would reinforce our conclusion. Indeed Keynesian features would partly kill the negative wealth effect associated with public expenditures, thus calling for an even higher degree of Edgeworth complementarity, all the more so as policy is countercyclical. We now briefly describe the augmented framework, then document our estimation strategy, and finally comment our empirical results.

3.1 The Model

The representative household’s intertemporal expected utility is

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ e^{c_{t+i}} \log(c_{t+i} + \alpha_g g_{t+i}) - e^{b_{t+i}} \frac{\eta}{1 + \nu} \left( \frac{n_{t+i}}{n_{t+i-1}} \right)^{1+\nu} \right\} \tag{14}$$

where $E_t \{ \cdot \}$ denotes the expectation operator conditional on the information set at period $t$ and $\beta \in (0, 1)$ is the subjective discount factor. As in the previous section, the parameter $\alpha_g$ governs the substitutability/complementarity between private consumption and public expenditures.
The parameter $\phi$ governs the habit persistence in labor supply and $\eta \geq 0$ is a scale parameter. When the parameter $\phi \neq 0$, labor supply decisions are subject to time non-separabilities. If $\phi < 0$, labor supply displays inter-temporal substitutability, whereas $\phi > 0$ implies inter-temporal complementarity. Martin Eichenbaum, Lars Peter Hansen & Kenneth J Singleton (1988) showed that a specification with intertemporal complementarities is favoured by the data. More recently, this specification has proven to be empirically relevant, as it translates habit persistence in leisure choices into aggregate output persistence (see Yi Wen 1998, Hafedh Bouakez & Takashi Kano 2006, Martial Dupaigne, Patrick Feve & Julien Matheron 2007). While other specifications that allow to capture the persistence in hours have been considered in the literature (e.g. adjustment costs on labor input, as in Yongsung Chang, Taeyoung Doh & Frank Schorfheide 2007, or learning-by-doing, as in Yongsung Chang, Joao F. Gomes & Frank Schorfheide 2002), it turns out that the implied reduced-form are almost identical to that resulting from our specification.

Utility derived from consumption is altered by a preference shock $a_t$ which obeys

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t}$$

where $|\rho_a| < 1$, $\sigma_a > 0$ and $\epsilon_{a,t}$ is iid with $\epsilon_{a,t} \sim N(0, 1)$. Labor disutility is subject to a preference shock $b_t$ which obeys

$$b_t = \rho_b b_{t-1} + \sigma_b \epsilon_{b,t}$$

where $|\rho_b| < 1$, $\sigma_b > 0$ and $\epsilon_{b,t}$ is iid with $\epsilon_{b,t} \sim N(0, 1)$. As noted by Jordi Galí (2005), this shock accounts for a sizeable portion of aggregate fluctuations. Moreover, it allows us to capture various distortions on the labour market, labeled labor wedge in V. V. Chari, Patrick J. Kehoe & Ellen R. McGrattan (2007).

The representative household supplies hours $n_t$ and capital $k_t$ to firms, and pays a lump-sum tax $T_t$ to the government. Accordingly, the representative household’s budget constraint in every period $t$ is

$$c_t + x_t \leq w_t n_t + r_t k_t - T_t$$

where $w_t$ is the real wage, $r_t$ is the rental rate of capital, and $x_t$ denotes investment. The capital stock evolves according to

$$k_{t+1} = (1 - \delta)k_t + x_t$$

where $\delta \in (0, 1)$ is the constant depreciation rate. The representative household thus maximises (14) subject to the sequence of constraints (15) and (16), $t \geq 0$.

The representative firm produces a homogeneous final good $y_t$ through the constant returns-to-scale technology

$$y_t = k_t^\theta (e^{z_t} n_t)^{1-\theta}$$

where $k_t$ and $n_t$ denote the inputs of capital and labour, respectively, $\theta \in (0, 1)$ is the elasticity of output with respect to capital, and $z_t$ is a shock to total factor productivity, which follows a random walk process with drift of the form

$$z_t = \log(\gamma_z) + z_{t-1} + \sigma_z \epsilon_{z,t}$$

where $\sigma_z > 0$ and $\epsilon_{z,t}$ is iid with $\epsilon_{z,t} \sim N(0, 1)$. The constant term $\gamma_z > 1$ is the drift term and accounts for the deterministic component of the growth process. Profit maximization equalizes the marginal productivity of each input factor to their price, so that $r_t = \theta y_t / k_t$ and $w_t = (1 - \theta) y_t / n_t$.

Government spending is entirely financed by taxes,

$$T_t = g_t$$
Notice that Ricardian equivalence holds in our setup, so that introducing government debt is unnecessary. The cyclical component of government spending is given by the policy rule

\[ g_t e^{-z_t} = \bar{g}^s y_t e^{\phi^s}; \]

where \( \bar{g}^s \) denotes the deterministic steady-state value of \( g_t e^{-z_t} \). The endogenous policy component \( \bar{y}_t \) obeys

\[ \log(\bar{y}_t) = -\varphi_g (\Delta \log(y_t) - \log(\gamma_z)); \]

and the stochastic (discretionary) component is assumed to follow an autoregressive process of the form:

\[ g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{g,t} \]

where \( |\rho_g| < 1, \sigma_g > 0 \) and \( \epsilon_{g,t} \) is iid with \( \epsilon_{g,t} \sim N(0,1) \). Here, \( \phi \) stands for the first-difference operator. The parameter \( \varphi_g \) is the policy rule parameter that links the cyclical component of government policy to demeaned output growth. Provided \( \varphi_g > 0 \), the policy rule features a countercyclical component that triggers an increase in government expenditures whenever output growth is below its average value. Nevertheless, the overall level of government spending need not be countercyclical since it incorporates the stochastic trend in productivity.

The homogeneous good can be used for private consumption \( c_t \), government consumption \( g_t \), and investment \( x_t \).

The market clearing condition on the good market accordingly writes

\[ y_t = c_t + x_t + g_t. \]

In the context of this model featuring a stochastic trend in productivity, we must modify Definition 1. To this end, we start by defining the detrended variables \( y_t^* = y_t e^{-z_t} \) and \( g_t^* = g_t e^{-z_t} \).

**Definition 2.** The long-run government spending multiplier, denoted by \( \Delta y / \Delta g \), is the increase in steady-state, detrended output \( \bar{y}^s \) after an increase in steady-state, detrended government spending expenditures \( \bar{g}^s \), i.e. formally

\[ \frac{\Delta y}{\Delta g} = \frac{d\bar{y}^s}{d\bar{g}^s}. \]

Using this definition, we can characterise how \( \alpha_g \) and the long-run GSM are linked together. This relation is stated in the following proposition, which generalises Proposition 1 to a setup with investment.

**Proposition 3.** Under the preceding assumptions:

1. The long-run government spending multiplier \( \Delta y / \Delta g \) is

\[ \frac{\Delta y}{\Delta g} = \frac{1 - \alpha_g}{1 - s_x + \mu[1 - s_x - s_g(1 - \alpha_g)]}, \]

where

\[ \mu \equiv (1 + \nu)(1 - \phi) - 1, \quad s_x \equiv \frac{(\gamma_z - 1 + \delta)\beta\gamma}{\gamma_z - \beta(1 - \delta)}. \]

2. The multiplier \( \Delta y / \Delta g \) decreases with \( \alpha_g \).

**Proof.** See Appendix C.

Notice that when \( \alpha_g \) and \( \phi \) are set to zero, the above formula collapses to those reported in Baxter & King (1993) and Aiyagari, Christiano & Eichenbaum (1992). As in the previous section, \( \varphi_g \) clearly
does not show up in this formula. It remains to be seen whether omitting $\varphi_g$ at the estimation stage can compromise the inference on $\varphi_g$.

3.2 Data and Estimation

Before taking the model to the data, we first induce stationarity by getting rid of the stochastic trend component $z_t$ and we log-linearize the resulting system in the neighborhood of the deterministic steady state. Then, let $\tilde{\xi}_t$ denote the vector collecting the log-linear model variables. The log-linear solution is of the form

$$\tilde{\xi}_t = F(\psi)\tilde{\xi}_{t-1} + G(\psi) \left( \begin{array}{c} \epsilon_{z,t} \\ \epsilon_{a,t} \\ \epsilon_{b,t} \\ \epsilon_{g,t} \end{array} \right),$$

(17)

where $\psi$ is the vector of model's parameters. The system matrices $F(\psi)$ and $G(\psi)$ are complicated functions of the model's parameters.

We use as observable variables in estimation the logs of output, consumption, hours worked, and government expenditures. The measurement equation is

$$\begin{pmatrix} \Delta \log(y_t) \\ \log(n_t) \\ \Delta \log(c_t) \\ \Delta \log(g_t) \end{pmatrix} = \begin{pmatrix} \gamma_z - 1 \\ m_n(\psi) \\ \gamma_z - 1 \\ 0 \end{pmatrix} + H\tilde{\xi}_t.$$

(18)

Here, $m_n(\psi)$ is a function that gives average log hours as a function of $\psi$ and $H$ is a selection matrix. For a given $\psi$, using equations (17) and (18), the log-likelihood is evaluated via standard Kalman filter techniques. The estimated parameters are then obtained by maximising the log-likelihood.

The data used for estimation come from the Federal Reserve Bank of St. Louis' FRED II database and from the Bureau of Labor Statistics website. They consist of government consumption expenditures and gross investment (GCE), private investment and private consumption, all deflated by the implicit GDP deflator (GDPDEF). Private investment is defined as the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). Private consumption is measured as the sum of personal consumption expenditures on non-durable goods (PCND) and services (PCESV). Output is then defined as the sum of private investment, private consumption and government expenditures. Hours are borrowed from Neville Francis & Valerie A. Ramey (2009). These hours data refer to the total economy and are adjusted for low-frequency movements due to changes in demographics, thus displaying less low-frequency behaviour than unadjusted data. All the series are converted to per-capita terms by dividing them by the civilian population, age 16 and over (CNP16OV). All the series are seasonally adjusted except for population. Our sample runs from 1960:1 to 2007:4.

The vector of parameters $\psi$ is split in two subvectors $\psi_1$ and $\psi_2$. The first one, $\psi_1 = (\beta, \delta, \nu, \theta, s_g)$, contains parameters calibrated prior to estimation. Typically, these are parameters difficult to estimate in our framework. The subjective discount factor, $\beta$, is set to 0.9951, yielding a real annual interest rate of 3.75%. The depreciation rate, $\delta$, is set to 0.0153, to match the average investment–output ratio. The parameter $\nu$ is set to 4 so that the long-run labour supply elasticity $\mu \equiv (1 + \nu)(1 - \phi) - 1$ is close to 2 in the benchmark model, in accordance with previous studies (Frank Smets & Rafael Wouters 2007). Finally we set $\theta = 0.30$, so that the labour income share in output is 70%, and $s_g = 0.2$, so as to reproduce the average ratio of government expenditures to output in our sample.

8 - See Appendix C for further details on the procedure used to induce stationarity.

9 - We used different measurement equations, using the logged private consumption–output and logged government expenditures–output ratios instead of consumption growth and government expenditures growth. Estimation results were almost identical.
The remaining parameters, contained in \( \psi_2 = (\phi, \alpha_g, \gamma, \varphi_g, \rho_g, \rho_a, \rho_b, \sigma_z, \sigma_g, \sigma_a, \sigma_b) \), are estimated. Estimation results are reported in Table 1. We consider four model restrictions, according to whether \( \alpha_g \) and \( \varphi_g \) are constrained. The table reports the log–likelihood \( \mathcal{L} \) for each model specification, which we use naturally as our selection criterion. The restrictions are summarised below.

- **Model (1):** \( \alpha_g = 0, \varphi_g = 0 \), so that \( g \) has no direct effect on the marginal utility of private consumption and is exogenous.
- **Model (2):** \( \alpha_g = 0, \varphi_g \neq 0 \), so that \( g \) has no direct effect on the marginal utility of private consumption and is endogenous.
- **Model (3):** \( \alpha_g \neq 0, \varphi_g = 0 \), so that \( g \) has a direct effect on the marginal utility of private consumption and is exogenous.
- **Model (4):** \( \alpha_g \neq 0, \varphi_g \neq 0 \), so that \( g \) has a direct effect on the marginal utility of private consumption and is endogenous.

Overall, the model specifications yield precisely estimated parameters. Several general comments can be made.

Table 1. Estimation Results

<table>
<thead>
<tr>
<th>Specification (1)</th>
<th>Specification (2)</th>
<th>Specification (3)</th>
<th>Specification (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_g = 0, \varphi_g = 0 )</td>
<td>( \alpha_g = 0, \varphi_g \neq 0 )</td>
<td>( \alpha_g \neq 0, \varphi_g = 0 )</td>
<td>( \alpha_g \neq 0, \varphi_g \neq 0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification (1)</th>
<th>Specification (2)</th>
<th>Specification (3)</th>
<th>Specification (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_g )</td>
<td>0.4004 (0.0764)</td>
<td>0.4138 (0.0730)</td>
<td>0.3706 (0.0714)</td>
<td>0.4110 (0.0638)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1.0043 (0.0007)</td>
<td>1.0043 (0.0007)</td>
<td>1.0043 (0.0007)</td>
<td>1.0044 (0.0007)</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.9469 (0.0087)</td>
<td>0.9592 (0.0079)</td>
<td>0.9535 (0.0079)</td>
<td>0.9756 (0.0055)</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>0.9725 (0.0049)</td>
<td>0.9775 (0.0047)</td>
<td>0.9795 (0.0047)</td>
<td>0.9834 (0.0031)</td>
</tr>
<tr>
<td>( \rho_b )</td>
<td>0.8346 (0.0424)</td>
<td>0.8345 (0.0411)</td>
<td>0.8366 (0.0396)</td>
<td>0.8399 (0.0330)</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>0.0107 (0.0005)</td>
<td>0.0107 (0.0005)</td>
<td>0.0107 (0.0005)</td>
<td>0.0107 (0.0005)</td>
</tr>
<tr>
<td>( \sigma_g )</td>
<td>0.0130 (0.0007)</td>
<td>0.0120 (0.0006)</td>
<td>0.0129 (0.0006)</td>
<td>0.0119 (0.0006)</td>
</tr>
<tr>
<td>( \sigma_a )</td>
<td>0.0103 (0.0009)</td>
<td>0.0102 (0.0009)</td>
<td>0.0123 (0.0011)</td>
<td>0.0133 (0.0013)</td>
</tr>
<tr>
<td>( \sigma_b )</td>
<td>0.0270 (0.0014)</td>
<td>0.0266 (0.0014)</td>
<td>0.0273 (0.0015)</td>
<td>0.0266 (0.0014)</td>
</tr>
</tbody>
</table>

\( \mathcal{L} \)

2655.3227
2679.4375
2665.9388
2701.3173


First, except for \( \rho_g \) and \( \sigma_g \), most parameters are pretty invariant to the restrictions imposed on \( \alpha_g \) or \( \gamma_g \). The log–likelihood comparison suggests the following comments. First, comparing specifications (1) with (3) or (2) with (4), one can clearly see that the restrictions \( \alpha_g = 0 \) is strongly rejected by the data. The P–values of the associated likelihood ratio tests are almost zero. In specifications (3) and (4), the parameter \( \alpha_g \) is negative and significantly different from zero, consistent with the above likelihood ratio tests. This result is not an artifact of allowing for an endogenous government policy. This suggests that private and public consumption are complements. This result echoes findings by Georgios Karras (1994). Bouakez & Rebei (2007) and Samah Mazraani (2010) also reach similar conclusions in a slightly different specification for the interaction of public and private consumption.

10 - The parameter vector \( \psi_2 \) also contains \( \bar{\eta} \), which is the average level of log hours. In our setup, estimating \( \bar{\eta} \) is equivalent to estimating \( \eta \). This parameter does not play any role in the log–linearized dynamics and is not reported.

11 - We numerically checked the estimation convergence by shocking initial conditions on parameter values in the likelihood maximization step. Upon convergence, we also plotted slices of the likelihood function around each estimated parameter value to check local identification.

12 - We also used a model version with a CES specification for the interaction of government spending and private consumption in utility, as these authors do. As discussed above, since both representations yield the same reduced form, this model version yields the exact same estimation results. However, the CES specification raises an identification problem that our specification eschews. It turns out that the government spending weight in utility is not separately identified in the CES case.
Second, the restriction $\gamma_g = 0$ is clearly rejected by the data. To see this, compare specifications (1) and (2) or (3) and (4). In both cases, the associated $P$-values are almost zero. These results strongly support the view that government policy comprises an endogenous component. To sum up, specification (4) is our preferred model. This specification thus features (i) a positive effect of government spending on the marginal utility of private consumption and (ii) a countercyclical feedback effect in government spending.\(^{13}\)

Even though all second order moments are given the same weight in the likelihood, comparing unconditional moments from the models to their empirical counterparts is useful to get some intuition why model (4) is preferred. Results are reported in table 2. We consider moments documenting the volatility, persistence, and co-movement of key variables.

All the model versions perform equally well in terms of fitting standard errors of key aggregate variables. More interestingly, we see that whether or not $\alpha_g = 0$ is imposed, the correlation between changes in private consumption and changes in government spending is smaller when $\gamma_g > 0$. To see this, compare the results for specifications (1) against (2) or (3) against (4). This illustrates our claim that in a standard neoclassical growth model, allowing for a countercyclical government spending policy works toward reducing the correlation between consumption growth and government spending shocks. Similarly, comparing the same specifications, we see that relaxing the constraint $\gamma_g = 0$ decreases the correlation between output growth and government expenditures growth. Conversely, whether or not $\gamma_g = 0$, relaxing the constraint $\alpha_g = 0$ increases the correlation between consumption growth and government spending growth. To see this, compare the results for specifications (1) against (3) or (2) against (4). This once again illustrates how Edgeworth complementarity and countercyclical policy interact in our model. Finally, we also see that, irrespective of constraints imposed on $\alpha_g$, the restriction $\gamma_g = 0$ deteriorates the model's ability to capture the persistence of changes in government expenditures.

Table 2. Moments Comparison

<table>
<thead>
<tr>
<th>Date</th>
<th>Specification (1)</th>
<th>Specification (2)</th>
<th>Specification (3)</th>
<th>Specification (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>0.0903</td>
<td>0.0093</td>
<td>0.0093</td>
<td>0.0095</td>
</tr>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>0.0050</td>
<td>0.0071</td>
<td>0.0071</td>
<td>0.0074</td>
</tr>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>0.0227</td>
<td>0.0255</td>
<td>0.0281</td>
<td>0.0255</td>
</tr>
<tr>
<td>$\sigma(\Delta y, \Delta c)$</td>
<td>0.0110</td>
<td>0.01170</td>
<td>0.0145</td>
<td>0.0068</td>
</tr>
<tr>
<td>$\rho(\Delta y)$</td>
<td>0.0066</td>
<td>0.0099</td>
<td>0.0068</td>
<td>0.0070</td>
</tr>
<tr>
<td>$\rho(\Delta y, \Delta c)$</td>
<td>0.3200</td>
<td>0.1165</td>
<td>0.1226</td>
<td>0.1116</td>
</tr>
<tr>
<td>$\rho(\Delta c)$</td>
<td>0.2482</td>
<td>0.0217</td>
<td>0.0232</td>
<td>0.0176</td>
</tr>
<tr>
<td>$\rho(\Delta x)$</td>
<td>0.2057</td>
<td>0.1599</td>
<td>0.0413</td>
<td>0.1425</td>
</tr>
<tr>
<td>$\rho(\Delta y, \Delta c)$</td>
<td>0.0948</td>
<td>-0.0164</td>
<td>0.1076</td>
<td>-0.0139</td>
</tr>
<tr>
<td>$\rho(\Delta n)$</td>
<td>0.3886</td>
<td>0.3299</td>
<td>0.5437</td>
<td>0.3100</td>
</tr>
<tr>
<td>$\text{corr}(\Delta y, \Delta c)$</td>
<td>0.5115</td>
<td>0.6236</td>
<td>0.6461</td>
<td>0.7282</td>
</tr>
<tr>
<td>$\text{corr}(\Delta y, \Delta x)$</td>
<td>0.9043</td>
<td>0.8025</td>
<td>0.8567</td>
<td>0.7315</td>
</tr>
<tr>
<td>$\text{corr}(\Delta c, \Delta x)$</td>
<td>0.2013</td>
<td>0.5616</td>
<td>0.3354</td>
<td>0.5873</td>
</tr>
<tr>
<td>$\text{corr}(\Delta c, \Delta y)$</td>
<td>0.2388</td>
<td>0.1909</td>
<td>0.2004</td>
<td>0.4976</td>
</tr>
<tr>
<td>$\text{corr}(\Delta c, \Delta n)$</td>
<td>-0.0288</td>
<td>0.1719</td>
<td>0.0209</td>
<td>0.0213</td>
</tr>
</tbody>
</table>

Notes: Sample period: 1960:1–2007:4. $\sigma(\cdot)$, $\rho(\cdot)$, and $\text{corr}(\cdot, \cdot)$ stand for standard deviation, first-order autocorrelation coefficient, and correlation, respectively. $\Delta$ is the first difference operator, $y$ is output, $c$ is consumption, $x$ is investment, $g$ is government expenditures, $n$ is hours worked.

We complement the above results by performing specification tests for the innovation of each variables used for estimation in equation (18), i.e. $\Delta \log(y_t)$ output growth, $\log(n_{t-1})$ the log of hours, $\Delta \log(c_t)$ private consumption growth, and $\Delta \log(g_{t-1})$ government consumption growth. The innovations are obtained as the difference between the observed variables and their predicted value at convergence of the estimation stage. The specification tests, reported in table 3, are conducted for the four model's specifications. The first column reports the Samuel S. Shapiro & Martin B. Wilk (1965) test statistic. The null hypothesis being tested is that the innovation of the variables listed on the left is normally distributed. A small value of the test statistic indicates a

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13 - In appendix D, we discuss the robustness of our estimation results, allowing successively for habits in consumption, dynamic adjustment costs on investment, alternative feedback policy rules, and news shocks on government spending rule.
rejection of the null, whereas a value close to unity favours the normality assumptions. On the right, we report the $P$-value (in %) of the test statistic. Except for consumption growth, normality is rejected in all cases. However, rejection is essentially driven by a few outliers. Given the parametric parsimony of the model, such a rejection is hard to interpret. More interestingly, table 3 also includes serial correlation tests. We report the least-squares coefficient obtained by projecting each innovation on its own lag. For each coefficient, we report the associated 95% confidence interval. We find that omitting the feedback rule deteriorate the results. Indeed, comparing specification (3) and (4) shows that consumption and government spending innovations display less serial correlation when the policy rule coefficient is not constrained to zero. Having explored the empirical properties of model (4), we now use this version to investigate the quantitative effects of omitting the government feedback rule.

3.3 Quantitative Implications for the Multiplier

In this section, we assess the quantitative impact of omitting the countercyclical component of government expenditures policy. We proceed in two steps. First, guided by the analytical results obtained in the previous section, we investigate the consequences for the long–run GSM of mis-specifying the policy rule. Second, we illustrate that these effects are also present in an assessment of shorter term multipliers.

Table 3. Specification Tests

<table>
<thead>
<tr>
<th>Specification</th>
<th>Normality</th>
<th>Serial Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shapiro–Wilk Statistic</td>
<td>$P$-value (in %)</td>
</tr>
<tr>
<td>(1) $\Delta \log(\mu_t)$</td>
<td>0.9657</td>
<td>0.0631</td>
</tr>
<tr>
<td>$\log(n_t)$</td>
<td>0.9747</td>
<td>0.3513</td>
</tr>
<tr>
<td>$\Delta \log(c_t)$</td>
<td>0.9847</td>
<td>7.2771</td>
</tr>
<tr>
<td>$\Delta \log(g_t)$</td>
<td>0.9668</td>
<td>10.0148</td>
</tr>
<tr>
<td>(2) $\Delta \log(\mu_t)$</td>
<td>0.9668</td>
<td>0.0673</td>
</tr>
<tr>
<td>$\log(n_t)$</td>
<td>0.9738</td>
<td>0.3577</td>
</tr>
<tr>
<td>$\Delta \log(c_t)$</td>
<td>0.9850</td>
<td>7.8055</td>
</tr>
<tr>
<td>$\Delta \log(g_t)$</td>
<td>0.9672</td>
<td>0.0741</td>
</tr>
<tr>
<td>(3) $\Delta \log(\mu_t)$</td>
<td>0.9655</td>
<td>0.0460</td>
</tr>
<tr>
<td>$\log(n_t)$</td>
<td>0.9704</td>
<td>0.4368</td>
</tr>
<tr>
<td>$\Delta \log(c_t)$</td>
<td>0.9800</td>
<td>10.7253</td>
</tr>
<tr>
<td>$\Delta \log(g_t)$</td>
<td>0.9699</td>
<td>1.4050</td>
</tr>
<tr>
<td>(4) $\Delta \log(\mu_t)$</td>
<td>0.9681</td>
<td>0.0097</td>
</tr>
<tr>
<td>$\log(n_t)$</td>
<td>0.9744</td>
<td>0.4346</td>
</tr>
<tr>
<td>$\Delta \log(c_t)$</td>
<td>0.9865</td>
<td>12.3860</td>
</tr>
<tr>
<td>$\Delta \log(g_t)$</td>
<td>0.9693</td>
<td>0.1211</td>
</tr>
</tbody>
</table>

Notes: Sample period: 1960:1–2007:4. $\Delta \log(\mu_t)$ denotes output growth, $\log(n_t)$ hours, $\Delta \log(c_t)$ private consumption growth, and $\Delta \log(g_t)$ government consumption growth. Specification (1): $\alpha_g = \gamma_g = 0$, Specification (2): $\alpha_g = 0 \neq \gamma_g = 0$, Specification (3): $\gamma_g \neq 0 \neq \gamma_g = 0$, Specification (4): $\alpha_g \neq 0 \neq \gamma_g \neq 0$. Coefficients are obtained by projecting each innovation on its own lag.

3.3.1 Long–Run Government Spending Multiplier

Upon inspecting models (3) and (4), we see that imposing $\gamma_g = 0$ strongly affects the estimated value of $\alpha_g$. When $\gamma_g$ is freely estimated, we obtain $\alpha_g = -0.95$ whereas we get $\alpha_g = -0.63$ when we impose $\gamma_g = 0$. Importantly, these parameter estimates are significantly different from each other, according to a standard Wald test. What does this imply for long–run government spending multiplier? To answer this question, we use the formula in Proposition 3 to estimate the long–run GSM. The estimated multipliers are reported in table 4, together with their standard errors. When $\gamma_g$ is restricted to zero, the estimated long–run multiplier is typically less than one, as obtains in standard models (see Baxter & King 1993, Aiyagari, Christiano & Eichenbaum 1992). Concretely, depending on the restrictions imposed on $\gamma_g$, we obtain values roughly comprised between 0.53 and 0.55.
Importantly, comparing specifications (1) and (2) in table 4, one can clearly see that $\gamma_g$ has almost no discernible effect on the long-run multiplier. The reason why is simple: the parameters $\beta$, $\delta$, and $\nu$ are restricted prior to estimation. In addition, the estimation results show that $\phi$ is relatively insensitive to the different specifications (see table 1). In our framework, omitting the feedback effect in the policy rule can impact on the long-run output multiplier only when the parameter $\alpha_g$ is freely estimated. This is the novel feature of our model, because the link between the policy feedback parameter ($\gamma_g$) and the degree of Edgeworth complementarity between public and private consumptions ($\alpha_g$) could obviously not be studied in frameworks imposing $\alpha_g = 0$. The columns associated with specifications (3) and (4) in table 4 thus give new results. More precisely, in model (3), the output multiplier is 0.97 while it reaches 1.31 in model (4). These values are significantly different from each other at conventional levels. As explained above, the higher multiplier in (4) derives from a smaller $\alpha_g$ than in (3).

To complement on these results, we consider the following exercise. We set the feedback parameters $\gamma_g$ to values on a grid between 0 and the estimated value obtained in specification (4), i.e. $\gamma_g = 0.6117$. For each value, all the remaining parameters in $\psi_2$ are re-estimated. The results are reported on figure 3.3.1.

Figure 2: Sensitivity to constraints on policy rule parameter $\gamma_g$

Notes: Sample period: 1960:1–2007:4. The feedback parameters $\gamma_g$ takes values on a grid between 0 and 0.6117. For each value, all the remaining parameters in $\psi_2$ are re-estimated. The upper left panel reports the log-likelihood as a function of $\phi_g$. The grey area corresponds to restrictions on $\phi_g$ that are not rejected at the 5% level according to a likelihood ratio test. This grey area is also reported in each of the other panels. The upper right panel reports the estimated value of $\alpha_g$ as a function of $\phi_g$. The bottom panels report the long-run multipliers on output and consumption.
The upper left panel reports the log–likelihood as a function of $\gamma_g$. The grey area corresponds to restrictions on $\gamma_g$ that are not rejected at the 5% level according to a likelihood ratio test. This grey area is also reported in each of the other panels in figure 3.3.1. The upper right panel reports the estimated value of $\alpha_g$ as a function of $\gamma_g$. The bottom panels report the long–run multipliers on output and consumption. The figure makes clear that even loose restrictions on $\gamma_g$ (i.e. restrictions not too far from the estimated value) are easily rejected and rapidly translate into higher $\alpha_g$ and much lower multipliers. Importantly, the continuous and decreasing mapping from $\gamma_g$ to $\alpha_g$ (and thus on long–run multipliers) echoes the analytical findings obtained in the simple model explored in the first section.

To sum up, there exists a strong interaction between the estimated values of $\gamma_g$ and $\alpha_g$ that have potentially dramatic implications for the quantitative assessment of the long–run government spending multiplier. These cannot be ignored if the model is to be used to assess recovery plans of the same size as those recently enacted in the US. The next section provides a quantitative illustration based on the ARRA stimulus package.

### 3.3.2 An Illustration with the US Fiscal Stimulus Package

So far, we have insisted on the long–run GSM. The main reason for this is that this number does not directly depend on the cyclicality of government policy. Focusing on this multiplier thus illustrates in an unambiguous way how mis-specifying the policy rule can result in a downward biased multiplier. However, all our results should in principle hold equally in an assessment of shorter term multiplier.

To illustrate this, we use the recent US Fiscal Stimulus Package and feed it into our model. To do so, we follow Uhlig (2010) and specify an autoregressive process to mimic the fiscal stimulus of the ARRA. More precisely, we assume that the discretionary component to policy follows the process which is initialized by setting $g_0^* = 0$ and $g_1^* = 0.32$, assuming that date $t = 1$ corresponds to 2009. As argued by Uhlig (2010), this process closely approximates the government spending path reported in Cogan et al. (2010) (Uhlig 2010, figure 1).

This process is fed into three model versions which we use to compute different versions of the ARRA multiplier. First, we use specification (3), which imposes an exogenous policy at the estimation stage. Second, we use our preferred specification (4), in which policy parameters are freely estimated. Finally, we freeze all non–policy parameters obtained in this specification and impose an exogenous policy rule (i.e. $\gamma_g = 0$). We do this to make sure that allowing for a countercyclical policy rule does not change the resulting short–run multiplier too much. The results are reported in figure 3.3.2. The thick dark line corresponds to the percent deviation of GDP from its steady state after the ARRA shock in specification (4). The dashed, dark line is the same impulse response obtained under an exogenous policy in specification (4). Finally, the dotted line is the GDP response obtained under specification (3). As the figure makes clear, with or without systematic policy, the GDP response is always much higher in specification (4) than in specification (3).

Over the whole ARRA horizon, the cumulated difference between the response obtained under specification (4) and that obtained under specification (3) is approximately 1.4% of steady–state US GDP. Now, to make things concrete, the extra government expenditures of the ARRA package amount to 8.8% of US, steady–state output (see Cogan et al. 2010). Thus, an econometrician using specification (3) would understate the cumulated output effect of the ARRA package by approximately 15.5% of the package’s size.
Figure 3: Stimulus Effect on Output

Notes: Estimated dynamic effects of government purchases in the February 2009 stimulus legislation on output, in relative deviation from steady state. Specification (3): $\alpha_g \neq 0$ and $\gamma_g = 0$; specification (4): $\alpha_g \neq 0$ and $\gamma_g \neq 0$; in specification (4) with $\gamma_g = 0$, we use the estimated value of $\alpha_g$ in (4) and impose the counterfactual $\gamma_g = 0$.

Notice finally that the exact value of $\gamma_g$ used in simulating specification (4) does not quantitatively affect the multiplier. This is consistent with our previous long–run findings. As argued before, this parameter plays a key role only at the estimation stage. Omitting it would lead an econometrician to downward bias the degree of Edgeworth complementarity, resulting in a seriously flawed assessment of the ARRA impact.

4. Conclusion

This paper proposed to quantitatively assess the consequences of mis-specifying the government spending rule on the estimated government spending multiplier within a DSGE framework. We first considered a simplified model version to show analytically that omitting the feedback rule at the estimation stage yields a downward–biased estimate of the long–run government spending multiplier. To establish this, we first showed that the multiplier is an increasing function of the degree of Edgeworth complementarity. In turn, complementarity and countercyclical policy interact through cross-equation restrictions, paving the way for a potential bias. We then estimated on post-war US data a quantitative model version and obtained that omitting the endogeneity of government spending exerts a severe, downward impact on the estimated long–run multiplier. Such a bias also characterises shorter–term multipliers and thus can seriously affect the quantitative assessment of fiscal packages such as the ARRA stimulus. Our results appear to be very robust to a series of perturbations to the benchmark specification.

In our framework, we have deliberately abstracted from relevant details in order to highlight, as transparently as possible, the empirical link between policy rule parameters and the degree of Edgeworth complementarity between private and public consumption. However, the recent literature insists on other modelling issues that might potentially affect our results. We mention three of them. First, allowing for a fraction of liquidity constrained agents and/or alternative specifications of agent’s preferences (Jordi Gali, J. David López-Salido & Javier Vallés 2007, Tommaso Monacelli & Roberto Perotti 2008) have proven to be useful mechanisms for reproducing the aggregate effect of government spending shocks, thus competing with the approach retained in this paper. It will be useful to investigate how such alternative specifications interact with the policy feedback rule. Second, as put forth by Leeper, Plante & Traum (2010), a more general specification of government spending rule, lump–sum transfers, and distortionary taxation is
needed to properly fit US data. This richer specification includes in addition to the automatic
stabilizer component, a response to government debt and co–movement between tax rates. An
important quantitative issue may be to assess which type of stabilization (automatic stabilization
and/or debt stabilization) interacts with the estimated degree of Edgeworth complementarity. Third,
Riccardo Fiorito & Tryphon Kollintzas (2004) have suggested that the degree of complementarity/
substitutability between government and private consumptions is not homogeneous over types of
public expenditures. This suggests to disaggregate government spending and inspect how feedback
rules affect the estimated degree of Edgeworth complementarity in this more general setup. These
three issues will constitute the object of further research.

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Appendix Not Intended for Publication

A. Proof of Proposition 1
To prove the first part of Proposition 1, evaluate equation (4) in the deterministic steady state, which implies

\[ \eta \dot{y} = \frac{1}{\bar{y} - (1 - \alpha_y) \bar{g}}. \]  

(A.1)

where \( \bar{y} \) and \( \bar{g} \) are the steady-state values of \( y \) and \( g \), respectively.

Total differentiation of the above equation then yields

\[ \nu d\bar{y} = -\frac{\bar{g}}{\bar{y} - (1 - \alpha_g) \bar{g}} d\bar{y} + \frac{(1 - \alpha_g) \bar{g}}{\bar{y} - (1 - \alpha_g) \bar{g}} d\bar{g}. \]  

(A.2)

Rearranging this expression and using Definition 1, one obtains finally

\[ \frac{d\bar{y}}{d\bar{g}} = \frac{1 - \alpha_g}{1 + \nu [1 - s_g(1 - \alpha_g)]}, \]  

(A.3)

as was to be shown.
To establish the second part of Proposition 1, differentiate the long-run GSM with respect to $g$

$$\frac{\partial}{\partial \alpha_g} \left( \frac{\Delta y}{\Delta g} \right) = -\frac{1 + \nu}{(1 + \nu|1 - s_g(1 - \alpha_g)|)^2} < 0.$$  

Thus $\Delta y/\Delta g$ decreases with $\alpha_g$, as was to be shown.

B. Proof of Proposition 2

B.1 Proof of Part 1

The proof of part 1 of Proposition 2 proceeds as follows. The reduced-form equations (7) and (8) rewrite

$$\hat{y}_t = \rho \hat{y}_{t-1} + \frac{\alpha}{1 + \alpha \varphi_g} u_t + \frac{1}{1 + \alpha \varphi_g} z_t, \quad (B.4)$$

$$\hat{y}_t = \rho \hat{y}_{t-1} + \frac{1}{1 + \alpha \varphi_g} u_t - \frac{\varphi_g}{1 + \alpha \varphi_g} \Delta z_t, \quad (B.5)$$

where $\Delta z_t = z_t - z_{t-1}$ and

$$\rho = \frac{\alpha \varphi_g}{1 + \alpha \varphi_g}.$$  

Provided $\alpha_g < 1$ and $\varphi_g \geq 0$, we have $\rho \in [0, 1)$, so that (B.4) and (B.5) display second-order stationarity.

Accordingly, they can be restated as

$$\hat{y}_t = \frac{1}{1 + \varphi_g} \sum_{i=0}^{\infty} \rho^i L^i (\alpha u_t + z_t), \quad (B.6)$$

$$\hat{y}_t = \frac{1}{1 + \varphi_g} \sum_{i=0}^{\infty} \rho^i L^i (u_t - \varphi_g \Delta z_t), \quad (B.7)$$

where $L$ is the backshift operator. Equation (B.7) can be reformulated as

$$\hat{y}_t = \frac{1}{1 + \alpha \varphi_g} \left[ \sum_{i=0}^{\infty} \rho^i L^i \right] u_t - \varphi_g \left( 1 + \frac{\rho - 1}{\rho} \sum_{i=1}^{\infty} \rho^i L^i \right) z_t \quad (B.8)$$

Combining (B.6) and (B.8), one obtains

$$\text{E}[\hat{y}] = \frac{1}{(1 + \alpha \varphi_g)^2} \left( \frac{\alpha \sigma_u^2}{1 - \rho^2} - \frac{\varphi_g \sigma_z^2}{1 + \rho} \right). \quad (B.9)$$

Then using the definition of $\rho$, (B.9) rewrites

$$\text{E}[\hat{y}] = \frac{(1 + \alpha \varphi_g) \alpha \sigma_u^2 - \varphi_g \sigma_z^2}{(1 + \alpha \varphi_g)(1 + 2 \alpha \varphi_g)}. \quad (B.10)$$

From (B.8), one also obtains

$$\text{E}[\hat{y}^2] = \frac{1}{(1 + \alpha \varphi_g)^2} \left( \frac{\sigma_u^2}{1 - \rho^2} + \frac{2 \varphi_g \sigma_z^2}{1 + \rho} \right). \quad (B.11)$$

Then using the definition of $\rho$, (B.11) rewrites

$$\text{E}[\hat{y}^2] = \frac{(1 + \alpha \varphi_g) \sigma_u^2 + 2 \varphi_g \sigma_z^2}{(1 + \alpha \varphi_g)(1 + 2 \alpha \varphi_g)}. \quad (B.12)$$
Now, combining (B.10), (B.12), and the definition of $\hat{\alpha}$ yields

$$\hat{\alpha} \equiv \frac{E[\hat{y}] - \alpha}{E[\hat{g}^2]} = \frac{(1 + \alpha \varphi_g)\sigma_u^2 - \varphi_g \sigma_z^2}{(1 + \alpha \varphi_g)\sigma_u^2 + 2\varphi_g^2 \sigma_z^2},$$

as was to be shown.

B.2 Proof of Part 2

The proof of part 2 of Proposition 2 is straightforward. Form the difference $\hat{\alpha} - \alpha$, which yields

$$\hat{\alpha} - \alpha = -\frac{\varphi_g \sigma_z^2 (1 + 2\alpha \varphi_g)}{(1 + \alpha \varphi_g)\sigma_u^2 + 2\varphi_g^2 \sigma_z^2}. \quad \text{(B.13)}$$

This expression is strictly negative whenever $\varphi_g > 0$ and $\sigma_z > 0$, as was to be shown.

B.3 Proof of Part 3

To prove the third part of proposition 2, differentiate (B.13) with respect to $\varphi_g$. Assuming $\sigma_z > 0$, this yields

$$\frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) = \frac{-1}{\omega (1 + \alpha \varphi_g) + 2\varphi_g^2} \cdot \frac{1}{\omega} \left( \frac{1}{(1 + \alpha \varphi_g)\sigma_u^2 + 2\varphi_g^2 \sigma_z^2} \right)^2 \omega^2 \varphi_g + \omega. \quad \text{(B.14)}$$

where we defined $\omega \equiv (\sigma_u / \sigma_z)^2$ and

$$P(\varphi_g) \equiv 2(\alpha^2 \omega - 1)\varphi_g^2 + 4\alpha \omega \varphi_g + \omega. \quad \text{(B.15)}$$

The expression in equation (B.14) is defined for $\varphi_g = 0$ whenever $\omega > 0$, i.e. whenever $\sigma_u > 0$. To simplify the analysis, we henceforth impose this condition. The sign of $\partial(\hat{\alpha} - \alpha) / \partial \varphi_g$ depends on the sign of $P(\varphi_g)$. In turn, the sign of $P(\varphi_g)$ depends on the sign of $\alpha^2 \omega - 1$, which calls for the following discussion:

- If $\alpha = \sigma_z / \sigma_u$, then $\alpha^2 \omega - 1 = 0$, and thus $P(\varphi_g) = 4\alpha \omega \varphi_g + \omega > 0$ for $\varphi_g \geq 0$. Hence
  $$\forall \varphi_g \geq 0, \frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) < 0.$$

- Now, if $\alpha > \sigma_z / \sigma_u$, then $\alpha^2 \omega - 1 > 0$, and thus $P(\varphi_g)$ admits two negative roots (as can be read from the coefficients). Thus, for $\varphi_g \geq 0, P(\varphi_g) > 0$. Hence
  $$\forall \varphi_g \geq 0, \frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) < 0.$$

- Finally, if $\alpha < \sigma_z / \sigma_u$, then $\alpha^2 \omega - 1 < 0$, and thus $P(\varphi_g)$ admits roots of opposite signs (as can be read from the coefficients). Let $\varphi_g(\alpha, \sigma_z, \sigma_u)$ define the positive root, i.e.
  $$\varphi_g(\alpha, \sigma_z, \sigma_u) = \frac{2\alpha \omega + \sqrt{2\omega(\alpha^2 \omega + 1) - 2(1 - \alpha^2 \omega)}}{2(1 - \alpha^2 \omega)} > 0.$$

Since the parabola opens downward, $P(\varphi_g) \geq 0$ for $\varphi_g \in [\varphi_g(\alpha, \sigma_z, \sigma_u)]$ and $P(\varphi_g) < 0$ for $\varphi_g > \varphi_g(\alpha, \sigma_z, \sigma_u)$. Thus

$$\frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) \leq 0, \quad \forall \varphi_g \in [0, \varphi_g(\alpha, \sigma_z, \sigma_u)],$$

$$\frac{\partial}{\partial \varphi_g} (\hat{\alpha} - \alpha) > 0, \quad \forall \varphi_g > \varphi_g(\alpha, \sigma_u, \sigma_z).$$

This completes the proof.
C. Proof of Proposition 3

To prove the first part of Proposition 3, we start by characterizing the deterministic steady state. The latter is defined in terms of detrended variables. To be more specific, we induce stationarity according to the following formulas:

\[ c_t^s = c_0 e^{-zt}, \quad x_t^s = x_1 e^{-zt}, \quad g_t^s = g_0 e^{-zt}, \quad y_t^s = y_1 e^{-zt}, \quad k_{t+1}^s = k_{t+1} e^{-zt}. \]

Using these definitions and the equilibrium conditions, the model steady state is solution to the system of equations:

\[ \begin{align*}
\bar{c}^s + \bar{x}^s + \bar{y}^s &= \bar{y}^s, \\
\bar{y}^s &= \left( \frac{\bar{k}^s}{\gamma_z} \right)^\theta \bar{n}^{1-\theta}, \\
\frac{1}{\bar{c}^s + \alpha_y \bar{y}^s} (1 - \theta) \bar{y}^s &= \eta (1 - \beta \phi) \bar{n}^{(1+\nu)(1-\phi)}, \\
1 &= \frac{\beta}{\gamma_z} \left( 1 - \delta + \frac{\theta \gamma_z \bar{y}^s}{\bar{k}^s} \right), \\
\bar{x}^s &= \left( 1 - \frac{1 - \delta}{\gamma_z} \right) \bar{k}^s.
\end{align*} \tag{C.16, C.17, C.18, C.19, C.20} \]

From (C.19) and (C.20), one can solve for the capital–output and investment–output ratios:

\[ \frac{\bar{y}^s}{\bar{k}^s} = \frac{\gamma_z - \beta (1 - \delta)}{\beta \theta \gamma_z}, \quad s_x = \frac{\bar{x}^s}{\bar{y}^s} = \frac{\beta \theta (\gamma_z - 1 + \delta)}{\gamma_z - \beta (1 - \delta)}. \]

It follows from these relations that \( s_x \) and \( \bar{y}^s/\bar{k}^s \) do not depend on \( \bar{y}^s \). Thus, the ratio \( \bar{y}^s/\bar{n} \) does not either depend on \( \bar{y}^s \).

Now, differentiating equation (C.16) with respect to \( \bar{y}^s \) yields:

\[ \frac{\partial \bar{c}^s}{\partial \bar{y}^s} = (1 - s_x) \frac{\partial \bar{y}^s}{\partial \bar{y}^s} - 1. \tag{C.21} \]

Differentiating equation (C.18), one obtains:

\[ (s_c + \alpha_y s_y) \frac{\partial \bar{y}^s}{\partial \bar{y}^s} = (1 + \nu) (1 - \phi) (s_c + \alpha_y s_y) \frac{\bar{n}}{\bar{y}^s} \frac{\partial \bar{n}}{\partial \bar{y}^s} + \frac{\partial \bar{c}^s}{\partial \bar{y}^s} + \beta \]

where

\[ s_c \equiv \frac{\bar{c}^s}{\bar{y}^s}. \]

Now, since the ratio \( \bar{y}^s/\bar{n} \) does not depend on \( \bar{y}^s \), it must be the case that:

\[ \frac{1}{\bar{n}} \frac{\partial \bar{n}}{\partial \bar{y}^s} = \frac{1}{\bar{y}^s} \frac{\partial \bar{y}^s}{\partial \bar{y}^s}, \]

so that the previous equation rewrites:

\[ (s_c + \alpha_y s_y) [1 - (1 + \nu)(1 - \phi)] \frac{\partial \bar{y}^s}{\partial \bar{y}^s} = \frac{\partial \bar{c}^s}{\partial \bar{y}^s} + \beta \].
Using equation (C.21) and Definition 1, one arrives at
\[
\frac{\Delta y}{\Delta g} = \frac{1 - \alpha_g}{1 - s_x + \mu[1 - s_x - s_g(1 - \alpha_g)]},
\]
where we made use of the identity \( s_c = 1 - s_x - s_g \).

To prove the second part of Proposition 3, differentiate the long-run GSM with respect to \( g \), which implies
\[
\frac{\partial}{\partial \alpha_g} \left( \frac{\Delta y}{\Delta g} \right) = -\frac{(1 - s_x)(1 + \mu)}{(1 - s_x + \mu[1 - s_x - s_g(1 - \alpha_g)])^2} < 0.
\]
This completes the proof.

**D. Robustness**

In this section, we offer a robustness analysis. We first resort to simulations to investigate small sample issues and to check whether our results are an artifact of our particular sample. Second, we consider extensions of our benchmark specification (4), allowing successively for habits and dynamic adjustment costs on investment, alternative feedback policy rules, and news shocks on government spending rule. All these additional modeling elements have received considerable attention in the recent literature and are thus worth considering. Finally, we reestimate the model on subsamples to check whether the relation between \( \alpha_g \) and \( \phi_g \) still holds.

**D.1 Results from Simulated Data**

Based on our previous results, one can suspect that the greater \( \alpha_g \) obtained under model (3) is the outcome of a misspecification bias. Indeed, we previously saw that omitting \( \phi_g \) always increases the estimated value of \( \alpha_g \).

In the simple model considered in the first section, we were able to formally show the existence of such a bias. In our DSGE framework, no such analytical results is available, though the same economic forces seem to be at play. To make our point, we thus resort to simulation techniques. Indeed, actual data are just one draw from an unknown DGP. Hence one cannot exclude that the negative link between \( \alpha_g \) and \( \phi_g \) is idiosyncratic to our sample. In addition, resorting to simulation enables us to investigate whether \( \phi_g \) can be estimated to non-zero values even in a world where no such mechanism exists (an exercise that we can hardly perform on actual data).

To investigate this, we develop a controlled experiment in which we use model (4) as our DGP, using the estimated values reported in table 1. More specifically, using model (4) as our DGP we first want to make sure that (i) estimating specification (4) on simulated data delivers consistent parameter estimates and (ii) estimating specification (3) on the exact same simulated data yields severely biased estimates of \( \alpha_g \). To complement on this, we also run the symmetric estimations in which we use model (3) as our DGP and successively estimate specifications (3) and (4) on simulated data. In this case, the crucial point is to check whether our estimation procedure is able to properly reject a policy feedback rule when no such rule exists in simulated data.\(^{15}\)

To begin with, table 5 reports the simulation results when using either specifications (4) or (3) as DGP and/or estimated model. Figure D.1 reports the empirical density of parameter estimates obtained from a Gaussian kernel. The dark line corresponds to the parameter distribution obtained by estimating model (4) on data simulated from model (4). The grey line corresponds to the parameter distribution obtained by estimating model (3) on data simulated from model (4). The red vertical line denotes the true value used for simulation. The dark vertical line is the average value obtained by estimating model (4) on data simulated from model (4). Finally, the grey vertical line is the average value obtained by estimating model (3) on data simulated from model (4).

\(^{15}\) In practice our Monte Carlo simulation are run as follows: using either model (4) or model (3), we generate 1000 samples of observables \( \Delta \log(y), \Delta \log(n), \Delta \log(c), \Delta \log(g) \), with the same sample size as actual data, after having eliminated 800 initial observations, thus ensuring that initial conditions do not contaminate our estimation results. To do so, the four structural shocks innovations are drawn from independent Gaussian distributions with zero mean and unit variance. On each simulated sample, we estimate specifications (3) and (4) and thus generate a population of estimated parameters.
Figure D.1 reports analog densities obtained when using model (3) as the DGP.

We first check whether estimating model (4) on data simulated from model (4) yields consistent parameter estimates. It turns out that this is the case. Indeed, we see from figure D.1 and table 5 that the average parameters estimates almost coincide with the true ones. Now, consider what happens when estimating model (3) on data simulated from model (4). In this case, all the parameters linked to government policy ($\alpha_g, \rho_g, \sigma_g$) turn out to be biased. This is particularly striking when it comes to $\alpha_g$, the average value of which is almost twice as small (in absolute term) as the true one. Figure D.1 once again offers a visual illustration of this. Interestingly, the average estimated value of $\alpha_g$ from our simulation experiment is very similar to what obtains from actual data when estimating model (3).

<table>
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<th>Table 5. Simulation Results</th>
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<td><strong>DGP: Specification (4)</strong></td>
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Notes: Simulation results obtained from 1000 replications. Model (4): $\alpha_g \neq 0$ and $\varphi_g = 0$; Model (3): $\alpha_g \neq 0$ and $\varphi_g = 0$. In each case, we report the average value of parameters across simulations.

Consider now what happens when using specification (3) as our DGP. The results are reported in table 5 and figure D.1. We first check whether estimating specification (3) on data simulated from model (3) yields consistent estimates. This again turns out to be the case. Now, consider what happens when estimating model (4) on data simulated from model (3). Basically, this procedure is able to recover the true parameters on average. This is particularly striking when it comes to the feedback parameter $\varphi_g$, which is zero on average. Recall that the latter does not exist in model (3), the DGP used for this simulation experiment, and appears only in model (4). This implies that a significant $\varphi_g$ on actual data does not seem to be an artifact of our particular sample.
Notes: Simulation results obtained from 1000 replications. The empirical density of parameter estimates is obtained from a Gaussian kernel. The dark line corresponds to the parameter distribution obtained by estimating model (4) on data simulated from model (4). The grey line corresponds to the parameter distribution obtained by estimating model (3) on data simulated from model (4). The red vertical line denotes the true value used for simulation. The dark vertical line is the average value obtained by estimating model (4) on data simulated from model (4). Finally, the grey vertical line is the average value obtained by estimating model (3) on data simulated from model (4).

Notes: Simulation results obtained from 1000 replications. The empirical density of parameter estimates is obtained from a Gaussian kernel. The dark line corresponds to the parameter distribution obtained by estimating model (3) on data simulated from model (3). The grey line corresponds to the parameter distribution obtained by estimating model (4) on data simulated from model (3). The red vertical line denotes the true value used for simulation. The dark vertical line is the average value obtained by estimating model (3) on data simulated from model (3). Finally, the grey vertical line is the average value obtained by estimating model (4) on data simulated from model (3).
D.2 Additional Real Frictions

A central ingredient of our preferred specification is the presence of dynamic complementarities in labor supply. Importantly, output dynamics inherit the built-in persistence of hours worked generated by this mechanism. The recent DSGE literature, however, has emphasized alternative real frictions capable of generating very strong aggregate persistence. Important such mechanisms are habits in consumption and dynamic investment adjustment costs, see Lawrence J. Christiano, Martin Eichenbaum & Charles L. Evans (2005). We considered versions of our preferred model augmented with these additional mechanisms.

When either of these are included, they do not significantly contribute to the model's fit. A standard likelihood ratio test would not reject the restriction of no habits in consumption and/or no dynamic adjustment costs. For example, in the case of specification (4) augmented with habits in consumption and dynamic adjustment costs, the log-likelihood is equal to 2702.22, to be compared to our reference specification (4) where the log-likelihood is equal to 2701.32. The habits in consumption parameter is equal to 0.11 (not significantly different from zero at conventional levels) and the adjustment cost parameter is almost zero. In addition, we redo the specification tests (normality and serial correlation). Including habits in consumption and dynamic adjustment costs does not improve upon the model performance: the normality test statistic is almost the same for each innovation and the serial correlation coefficients are very similar.

More importantly for our purpose, the empirical interaction between \( \alpha_g \) and \( \phi_g \) still holds under this more complete framework. In particular, when \( \phi_g \) is constrained to zero, we obtain \( \alpha_g = -0.21 \), yielding a multiplier \( \Delta y = \Delta g = 0.85 \). In contrast, when \( \phi_g \) is freely estimated, government policy turns out to be countercyclical (\( \phi_g = 0.60 \)) and the parameter \( \alpha_g = -0.79 \), implying a multiplier equal to 1.19. This confirms our main result.

D.3 Alternative Specifications of the Feedback Rule

We also experimented with alternative specifications for the government spending feedback rule. These alternative rules are specified as follows

(A) \( \log(\bar{y}_t) = -\varphi_g (\log(y_t) - z_t - \log(\bar{y}^*)) \)
(B) \( \log(\bar{y}_t) = -\varphi_g (\log(y_{t-1}) - z_{t-1} - \log(\bar{y}^*)) \)
(C) \( \log(\bar{y}_t) = -\varphi_e (\Delta z_t - \log(\gamma_z)) \)
(D) \( \log(\bar{y}_t) = -\varphi_e (\Delta z_t - \log(\gamma_z)) - \varphi_a \Delta a_t - \varphi_b \Delta b_t \)
(E) \( \log(\bar{y}_t) = -\varphi_g (\Delta \log(y_t) - \log(\gamma_z)) - \varphi_e (\Delta z_t - \log(\gamma_z)) \)
(F) \( \log(\bar{y}_t) = -\varphi_g (\Delta \log(y_t) - \log(\gamma_z)) - \varphi_e (\Delta z_t - \log(\gamma_z)) - \varphi_a \Delta a_t - \varphi_b \Delta b_t \)

(where, as before, \( \bar{y}^* \) denotes the steady-state value of detrended output. Results are reported in table 6. For comparison purpose, we reproduce the results obtained with our preferred specification (4), referred to here as the benchmark specification. The other specifications are: (A) the stationary component of government spending reacts to current deviations of output from its stochastic trend; (B) the stationary component of government spending reacts to once-lagged deviations of output from its stochastic trend; (C) the stationary component of government spending reacts to changes in total factor productivity, resembling the specification used by Smets & Wouters (2007); (D) the stationary component of government spending reacts to changes in all the structural shocks; (E) combines our benchmark specification with (C); (F) combines the benchmark specification with (D). Table 6 reports the estimated values of \( \alpha_g \) and the policy rule parameters, together with the implied long-run multipliers and the log-likelihood. To ease comparison, we also report the estimation results obtained under an exogenous policy (i.e. specification (3)).

As is clear from table 6, our benchmark specification dominates the alternative specifications from (A) to (D). Notice that these specifications are not nested with our benchmark. Since our benchmark implies a higher log-likelihood, the alternative rules are not better descriptions of the
data. Moreover, the specification test results deteriorate (not reported), especially so in terms of serial correlation of innovations.

### Table 6. Alternative Government Spending Rules

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\alpha_g$</th>
<th>$\varphi_g$</th>
<th>$\varphi_z$</th>
<th>$\varphi_a$</th>
<th>$\varphi_b$</th>
<th>$\phi$</th>
<th>$\Delta y/\Delta g$</th>
<th>$\mathcal{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>-0.9452</td>
<td>0.6117</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.4110</td>
<td>1.3128</td>
<td>2701.3173</td>
</tr>
<tr>
<td>(A)</td>
<td>-0.5774</td>
<td>-0.4866</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.4072</td>
<td>0.9652</td>
<td>2669.6339</td>
</tr>
<tr>
<td>(B)</td>
<td>-0.6287</td>
<td>0.1029</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3710</td>
<td>0.9623</td>
<td>2666.1522</td>
</tr>
<tr>
<td>(C)</td>
<td>-0.7091</td>
<td>-0.3952</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3995</td>
<td>1.0693</td>
<td>2691.5604</td>
</tr>
<tr>
<td>(D)</td>
<td>-0.7893</td>
<td>0.4254</td>
<td>0.0133</td>
<td>-0.0500</td>
<td>0.3769</td>
<td>1.1105</td>
<td>2694.4357</td>
<td></td>
</tr>
<tr>
<td>(E)</td>
<td>-1.0398</td>
<td>0.8176</td>
<td>-0.1660</td>
<td>-</td>
<td>0.4181</td>
<td>1.4244</td>
<td>2702.0942</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>-1.2220</td>
<td>2.5102</td>
<td>-1.4928</td>
<td>-0.1132</td>
<td>0.2264</td>
<td>0.3700</td>
<td>1.5455</td>
<td>2714.2522</td>
</tr>
<tr>
<td>Exogenous Policy</td>
<td>-0.6340</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3766</td>
<td>0.9738</td>
<td>2665.9338</td>
</tr>
</tbody>
</table>

Notes: Sample period: 1960:1–2007:4. Benchmark is specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$. Exogenous policy is specification (3): $\alpha_g \neq 0$ and $\varphi_g = 0$.

The specifications that yield the lowest fit are (A) and (B). Specification (A) implies a procyclical government spending policy. In this case, the implied $\alpha_g$ is higher than under an exogenous policy ($-0.58$ in specification (A) and $-0.63$ under an exogenous policy). This mechanically translates into a smaller multiplier in (A), as expected from our previous analysis. In specification (B), the policy rule is almost acyclical and we obtain roughly similar multipliers than under an exogenous policy.

Specification (C) implies a countercyclical policy rule in response to technology shocks, since the estimated values of $\varphi_g$ is positive. Once again, as expected, the estimated $\alpha_g$ is slightly lower than under an exogenous policy ($\alpha_g = -0.71$), resulting in a higher multiplier.

Specification (D) adds to the former case by allowing policy to respond to all the shocks. This specification implies a countercyclical policy rule, in the sense that the estimated values of $\varphi_z$ and $\varphi_a$ are positive while the estimated value of $\varphi_b$ is negative. As expected, the Edgeworth complementarity parameter turns out to be lower than under an exogenous policy ($\alpha_g = -0.79$).

Specifications (E) and (F), which nest our benchmark, imply higher log–likelihoods, by construction. A likelihood ratio test would not reject our benchmark when compared to specification (E). In contrast, the log–likelihood is much higher in specification (F). However, specification tests outcomes do not improve much when compared to our benchmark.

It is not obvious a priori to tell whether or not specifications (E) and (F) feature countercyclicality of government spending policy. However, if our claim holds in this context, one can interpret the low value obtained for $\alpha_g$ as suggestive of a very high degree of countercyclicality.

### D.4 News Shocks in the Government Spending Rule

As emphasized by Valerie A. Ramey (2009) and Stephanie Schmitt-Grohe & Martin Uribe (2008), the expected component in public expenditures constitutes an important element of government policy. We accordingly modify our benchmark specification to allow for news shocks in the government spending rule, according to

$$g_t^* = \rho g_{t-1}^* + \sum_{i=0}^{q} \sigma_{g,t} \varepsilon_{g,t-i}, \; \forall t \in \{0, \ldots, q\}, \sigma_{g,t} \geq 0,$$

where the $\varepsilon_{g,t}$ is iid with $\varepsilon_{g,t} \sim N(0, 1)$. 
We first imposed $q = 4$. According to our estimation results, we obtain that lags $i = 1, 2, 3$ are not significant. This specification delivers a significantly better fit to the data than our preferred model (4), according to the likelihood ratio test (in this case, the log–likelihood is equal to 2717.39). However, the parameter estimates do not change too much compared to specification (4). In particular, the parameter $\alpha_g$ is now equal to $-0.86$, whereas the feedback rule parameter is equal to 0.59. In addition, allowing for news shocks in government spending does not improve upon the specification tests of our reference model (4).

Importantly, adding news shocks does not modify our main conclusion. When policy is exogenous ($\varphi_g = 0$), we obtain $\alpha_g = -0.21$, with an associated multiplier equal to 0.56. In contrast, when $\varphi_g$ is freely estimated, $\alpha_g$ is smaller, resulting in a higher multiplier $\Delta y = \Delta g = 1.11$.

D.5 Subsample Analysis

Roberto Perotti (2005) showed that empirical measures of government spending multipliers can prove sensitive to the particular sample selected. Importantly, he argues that multipliers over the sample 1980:1–2001:4 are smaller than those found over the sample 1960:1–1979:4. We here investigate whether our results still hold if we re–estimate our model over the same subsamples.

Results are reported in table 7. Our previous conclusions are broadly confirmed. First, the restriction $\varphi_g = 0$ is rejected, suggesting that government spending policy is endogenous, irrespective of the selected sample. Second, when this restriction is imposed, we obtain a higher $\alpha_g$, resulting in a smaller multiplier. This holds over both subsamples. We also obtain a smaller long–run GSM over the second subsample, confirming results in Perotti (2005).

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\alpha_g$</th>
<th>$\varphi_g$</th>
<th>$\phi$</th>
<th>$\Delta y/\Delta g$</th>
<th>$\mathcal{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960:1-1979:4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>-1.0167</td>
<td>0.6033</td>
<td>0.3755</td>
<td>1.3263</td>
<td>1101.3585</td>
</tr>
<tr>
<td>(3)</td>
<td>-0.7910</td>
<td>–</td>
<td>0.3896</td>
<td>1.1276</td>
<td>1088.1722</td>
</tr>
<tr>
<td>(4)</td>
<td>-0.8068</td>
<td>0.5597</td>
<td>0.4146</td>
<td>1.1856</td>
<td>1614.9772</td>
</tr>
<tr>
<td>(3)</td>
<td>-0.6100</td>
<td>–</td>
<td>0.3777</td>
<td>0.9580</td>
<td>1599.7944</td>
</tr>
</tbody>
</table>

Notes: Specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Specification (3): $\alpha_g \neq 0$ and $\varphi_g = 0$. 
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